$T\bar{T}$ and Double Trace Deformations

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At GLSC2017, Herman Verlinde talked about 1611.03470

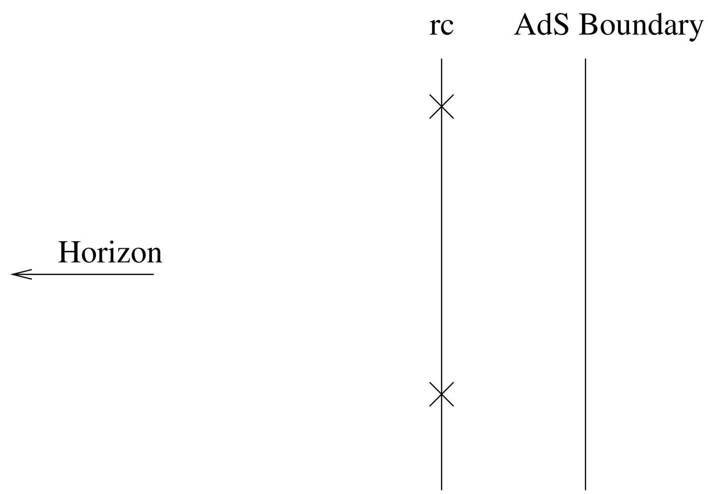
- $2d$ CFT's can finitely be deformed with $\mu \int d^2 z T(z)\bar{T}(\bar{z})$ (Smirnov and Zamolodchikov)

- Resulting system is UV complete (but is not a field theory)

- The spectrum of physical states (on a cylinder of size L) are cut-off at $E \sim M \sim 1/\tilde{\mu}$

$$E_n = \frac{2\pi}{\tilde{\mu}} \left(1 - \sqrt{1 - 2\tilde{\mu} M_n + \tilde{\mu}^2 J_n^2}\right), \qquad \tilde{\mu} = \frac{\pi \mu}{L^2}$$

- Interpret holographically as moving the boundary from $r = \infty$ ...



- ... to $r_c^2 = \frac{\ell_{ads}^4}{\mu c}$. Dirichlet Wall

- At GLSC2017, I talked about double trace deformations in AdS/CFT in the context of Gibbsian ruling in thermodynamics. (A lot of that is understanding renormalization scheme dependences) Natural to explore the relation.

- $T\bar{T}$ is a double trace deformation...

- but is slightly different from the story of Klebanov and Witten

- $T\bar{T}$ in $d = 2$ has dimension 4. It is *irrelevant*. It is in fact rather miraculous that one can perform a finite deformation of a CFT by an irrenevant operator at all. (Z, SZ, CNST) (Also Itzhaki...)

- KW considered alternate (minus) dressing in the AdS/CFT formula

$$\Delta_{\pm} = \frac{d \pm \sqrt{d^2 + 4m^2}}{2}$$

where $2\Delta_{-} < d$ (relevant) $\Rightarrow$ alternate CFT/alternate boundary condition (Neumann), deformed by a relevant operator, which subsequently flows to the usual (Dirichlet) CFT.

• The lowest irrelevant deformation characterizing the approach to the IR fixed point is a double trace operator of dimension $2\Delta_+$.

• Relation to SZ/MMV more natural as seen from the bottom, up.

• Consider a CFT which is solved. This means one is provided with the generating functional for all of the operators.

$$Z_D[g_{ij}^{\infty}(x), A_i^{\infty}(x), \phi^{\infty}(x), \ldots]$$

• AdS/CFT provides the same data.

$$\mathcal{O}(x) \leftrightarrow \frac{\delta}{\delta\phi^{\infty}(x)}, \qquad T^{ij}(x) \leftrightarrow \frac{\delta}{\delta g_{ij}^{\infty}(x)}, \qquad \ldots$$

Let's use these ingredients to contemplate what it might mean to deform the CFT with a double trace defomration of the form

$$\mu \int d^d x \, \mathcal{O}(x)\mathcal{O}(x)$$

This amounts to considering

$$Z_{def}[\phi^\infty(x), \ldots] = e^{-\mu \int d^d x \frac{\delta}{\delta \phi^\infty(x)} \frac{\delta}{\delta \phi^\infty(x)}} Z_D[\phi^\infty(x), \ldots]$$

This expresion admits a concrete interpretation using the following formal manipulations.

The idea is to insert

$$1 = \int [D\varphi] \delta(\phi(x) - \varphi(x)) = \int [DJ][D\varphi] e^{i \int d^d x \, J(x)(\phi^\infty(x) - \varphi^\infty(x))}$$

Then, we find

$$Z_{def}[\phi^\infty(x),\ldots] = \int [D\varphi^\infty] e^{\int d^dx\,\left(\frac{1}{4\mu}\varphi^\infty(x)^2 - \frac{1}{2\mu}\phi^\infty(x)\varphi^\infty(x) + \frac{1}{4\mu}\phi^\infty(x)^2\right)} Z_D[\varphi^\infty(x),\ldots]$$

• This is Legendre transform of Z_D deformed by a double trace deformation of dimension $2\Delta_-$ and a contact term deformation.

• Finite deformation by irrelevant operator of dimension $2\Delta_+$ admits a UV complete description in terms of a relevant deformation of alternate (Neumann) CFT.

• This should work whenever the Legendre transform makes sense.

• Can this be extended to spin 2 operators?

Before going all the way to spin 2, let's warm up by considering the story for spin 1

- $Z[A_i] \leftrightarrow$ CFT with $U(1)$ global symmetry sourcing operator J^i
- Consider Thirring deformation $\mu \int d^dx\, J_i(x) J^i(x)$
- Insert

$$1 = \int \frac{[Da_i]D[\sigma][DJ_i]}{\text{Vol(G)}} e^{i\int d^dx\,(a_i(x) - \partial_i\sigma(x) - A_i(x))J^i(x)}$$

- Path integral over a_i must be treated as a gauge field
- σ can be viewed as Stuckelberg field, or Lagrange multiplier enforcing $\partial_i J^i = 0$.

$$
\begin{aligned}
Z_{def}[A_i^\infty] &= e^{-\mu \int d^dx g_{ij}^\infty(x) \frac{\delta}{\delta A_i^\infty(x)} \frac{\delta}{\delta A_j^\infty(x)}} Z_D[A_i^\infty(x), \ldots] \\
&= \int \frac{[Da_i^\infty][D\sigma]}{\text{Vol}(G)} e^{\int d^dx \left(\frac{1}{4\mu} g_{ij}^\infty (a_i - \partial_i\sigma)(a_j - \partial_j\sigma) - \frac{1}{2\mu}(a_i - \partial_i\sigma) A^{\infty\, i} + \frac{1}{4\mu} A_i^\infty A^{\infty\, i} \right)} \\
&\qquad \times Z_D[a_i^\infty(x), \ldots]
\end{aligned}
$$

- “Gauging the global $U(1)$ of the original CFT” (Neumann)

- Deformation by double trace deformation

- Deformation not of Witten’s Chern-Simons type, but rather of the Marolf-Ross type

For spin 2, we should follow the same procedure of writing "1."

Instead we will go backwards stating the expected answer and discussing if it is interpretable as $T\bar{T}$ deformation or not.

- We are gauging $T \Rightarrow$ quantum gravity (Neumann)
- Turning on a relevant gauge invariant deformation

$$Z_{tensor}[\alpha] = \int \frac{[Dg_{ij}^{\infty}]}{\text{Vol(Diff)}} e^{-2\alpha \int d^d x \sqrt{g_\infty}} Z_D[g_{ij}^{\infty}(x), a_i^{\infty}(x), \phi^{\infty}(x)]$$

- No local observables
- Quantum gravity does make sense* in $2d$

Comments

- Different from MVV who assumed a Dirichlet wall

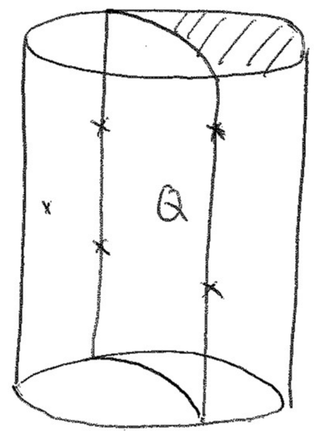
- Set of observables therefore different

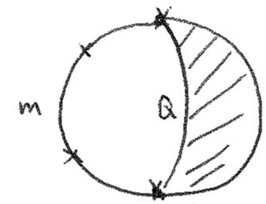
- Invoking quantum gravity consistent with the expectation that the UV complete descrition is not a field theory

- Matches with the expectation based on Randall-Sundrum/Karch-Randall constructions where a cut-off in UV degrees of freedom is associated with localizing gravity (though in 2d, there are no gravitons)

- In fact, Karch-Randall setup required a (positive) finite tension UV brane in order to localize Minkowski gravity. $\Leftrightarrow \alpha$

Detune the tension to induce a negative cosmological constant



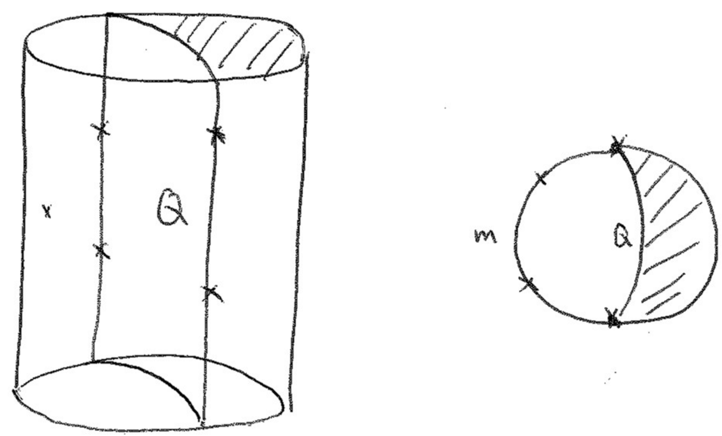


- There are no local observables on the world volume Q.

- But there can be local observables on ∂Q. In the flat space limit, this becomes the S-matrix

- Additional local observables on M if the original CFT admits a gravity dual.

It is interesting that from the perspective of M, this is the holographic BCFT of Takayanagi



- Quantum gravity on Q admits a dual descritpion as BCFT on M
- Mapping/interpretation of the observables seems complicated
- Only valid if the CFT we started with admits a gravity dual $c \gg 1$.

- Saying $2d$ gravity makes sense generically is too strong.
- Need to cancel Weyl anomaly.
- For $c \leq 1$, one can invoke Liouvlle.
- For $c > 26$, one can invoke Liouville too, but has strange signature.
- Convenient to work with critical ($c = 26$) long string extended along X_0 and X_1, embedded in $(25, 1)$ Minkowski space.
- Closed strings (including the bosonic tachyon) decouple for $g \to 0$ leaving $2D$ gravity.
- We are giving up having a useful holographic dual (and the BCFT picture) because $c = 26$ is not $\gg 1$.

- Chosing static gauge, one obtains a theory of 24 transverse scalars coupled to gravity in $2d$ with asymptotically Minkowski background geometry.

$$\frac{1}{\alpha'}\int d^2z\sqrt{-g}g^{ab}G_{IJ}\partial_a X^I\partial_b X^J \Rightarrow \int d^2z\frac{1}{\alpha'}\sqrt{-g}+\frac{1}{\alpha'}\sqrt{-g}g^{ab}G_{ij}\partial_a X^i\partial_b X^j$$

Here, α'^{-1} plays the role of the cosmological constant.

- Observables for this setup consists of S-matrix of 24 transverse scalars

- These have been computed by Dubovsky and collaborators. For $2 \rightarrow 2$:

$$S = \mathbf{1}e^{i\alpha' s}$$

- Interpretable as $T\bar{T}$ deformed scattering amplitudes

- Confirming our guess

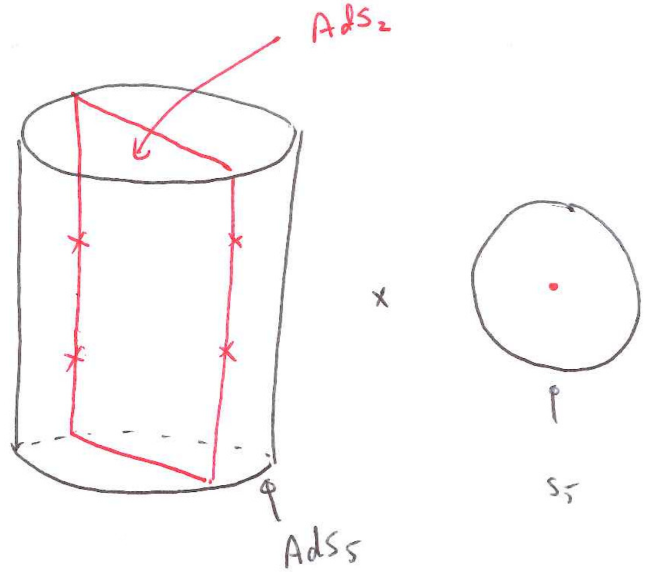
$$Z_{tensor}[\alpha] = \int \frac{[Dg_{ij}^{\infty}]}{\text{Vol(Diff)}} e^{-2\alpha \int d^dx \sqrt{g_\infty}} Z_D[g_{ij}^{\infty}(x), a_i^{\infty}(x), \phi^{\infty}(x)]$$

- That $T\bar{T}$ deformation of 24 scalars is Nambu-Goto-Polyakov is known, so this can't really be claimed as a new result, although thinking abstractly about 2d quantum gravity looks potentially useful

- Finding non-trivial generalizations are hard because examples where the $2d$ gravity is tractable is few and far in between: $c = 26$, $c = 1 +$ Liouville, $c = 1/2+$Liouvlle.

- Basing examples based on situations where a static long string can be arranged is a useful guiding principle

A neat example: Embed type II fundamental string in $AdS_5 \times S_5$ and sent $N \to \infty$ and $g_s \to 0$ keeping $g_s N$ fixed.



Worldsheet becomes AdS_2 with matter living in S_5 and transerse fluctuations in AdS_5. $\Rightarrow$ Quantum gravity on AdS_2 with matter.

We know what the observables should be: boundary deformations of 8 transverse scalars and their fermion partners

It is not easy to solve this string theory because RR fields are involved.

But we can compute them by interpreting these observables as expectation values of deformed Wilson loop in 't Hooft limit.

$$\langle \mathrm{Tr} P \left[(D_i(s_1) \ldots \Phi_j(s_2) \ldots) e^{\oint ds(A_0+\Phi_1)} \right] \rangle$$

Drukker et.al. (1703.03812) has shown that there are indeed 8 bosonic and 8 fermionic deformations exhibiting conformal scaling

This is AdS_2/CFT_1. Can take Minkowski limit (large gN)

Conclusions

• $T\bar{T}$ and related deformation has simple interpretations in terms of Legendre transforms and formal path integrals.

• Seemingly miraculous results by Zamolodchikov and others should have simple derivations based on this simple observation (or not.)

• There are some subtelties having to do with signs of vairous quantities such as μ, α', etc, which has important physical consequences which needs to be reviewed more carefully.

• So far, this perspective is most useful for crtictical long strings, which does not admit any holographic dual (in the sense of $AdS_3 \times CFT_2$). It would be nice to explore that.