# Excitations the Myers-Perry Geometry

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work in progress

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  - scalar fields in all dimensions
  - electromagnetic field and gravitons in 4D

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- Result: separation is controlled by eigenvectors of the Killing–Yano tensor

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- Summary

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$$l^\mu \partial_\mu = \frac{r^2 + a^2}{\Delta}\partial_t + \partial_r + \frac{a}{\Delta}\partial_\phi, \quad n^\mu \partial_\mu = \frac{r^2 + a^2}{2\Sigma}\partial_t - \frac{\Delta}{2\Sigma}\partial_r + \frac{a}{2\Sigma}\partial_\phi,$$

$$m^\mu \partial_\mu = \frac{1}{\sqrt{2}\rho}\left[ias_\theta \partial_t + \partial_\theta + \frac{i}{s_\theta}\partial_\phi\right], \quad \rho = r + iac_\theta, \ \Sigma = \rho\bar{\rho}, \ \Delta = r^2 + a^2 - 2Mr.$$

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$$F_{\mu\nu} = 2\left[\phi_1(n_{[\mu} l_{\nu]} + m_{[\mu}\bar{m}_{\nu]}) + \phi_2 l_{[\mu} m_{\nu]} + \phi_0 \bar{m}_{[\mu} n_{\nu]}\right] + cc.$$

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- Problems with the known solution:
  - the remaining eqn is not separable — Starobinsky–Churilov '73, Teukolsky–Press '74
  - it is hard to recover the gauge potential — Chandrasekhar '76
  - construction is based on $F$ and $\star F \Rightarrow$ hard to extend to $D > 4$

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- New ansatz may solve these problems and lead to extensions to $D > 4$

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$$l^\mu A_\mu = \frac{2ia}{r} l^\mu \partial_\mu [e^{i\omega t + im\phi} g_+(r) f_+(\theta)] + 2 l^\mu \partial_\mu H_+(r,\theta)$$

  - up to overall factors, no $\theta$ in $(l^\mu, n^\mu)$, no $r$ in $(m^\mu, \bar{m}^\mu)$

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- New proposal for a separable ansatz

$$l^\mu A_\mu = G_+(r) l^\mu \partial_\mu \Psi, \qquad n^\mu A_\mu = G_-(r) n^\mu \partial_\mu \Psi,$$
$$m^\mu A_\mu = F_+(\theta) m^\mu \partial_\mu \Psi, \quad \bar{m}^\mu A_\mu = F_-(\theta) \bar{m}^\mu \partial_\mu \Psi, \quad \Psi = e^{i\omega t + im\phi} R(r) S(\theta).$$

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- Most general separable solution of Maxwell's equations
  - functions $G_\pm$ and $F_\pm$ are completely determined by integrability conditions

$$l_\pm^\mu A_\mu = \pm \frac{ia}{r \pm i\mu a} \hat{l}_\pm \Psi, \quad m_\pm^\mu A_\mu = \mp \frac{1}{c_\theta \mp \mu} \hat{m}_\pm \Psi$$

  - Maxwell's equations $\Rightarrow$ "master equations" for $(S, R)$:

$$\frac{D_\theta}{s_\theta} \frac{d}{d\theta} \left[ \frac{s_\theta}{D_\theta} \partial_\theta S \right] + \left\{ -\frac{2\Lambda}{D_\theta} - (a s_\theta)^2 \left[ \omega + \frac{m}{a s_\theta^2} \right]^2 + \Lambda \right\} S = 0$$

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  - electromagnetism and scalar field are covered

scalar : $\quad D_r = 1, \quad D_\theta = 1, \quad \forall \Lambda;$

photon : $\quad D_r = 1 + \frac{r^2}{(\mu a)^2}, \quad D_\theta = 1 - \frac{c_\theta^2}{\mu^2}, \quad \Lambda = -\frac{1}{\mu} \left[ a\omega + m - a\omega\mu^2 \right]$

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- Can this construction be extended to $D > 4$ and other fields?

# Myers–Perry geometry

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- General properties
  - $\left[\frac{D-1}{2}\right]$ rotations in $D$ dimensions
  - different structures in odd and even $D$
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  - $D = 2(n+1)$: $U(1)_t \times [U(1)]^n$ isometry
- Separable Klein–Gordon & Dirac eqns $\Rightarrow$ families of Killing(–Yano) tensors

$$\nabla_\mu Y^{(k)}_{\nu_1 \ldots \nu_k} + \nabla_{\nu_1} Y^{(k)}_{\mu \ldots \nu_k} = 0, \quad Y^{(D-2k)} = \star \Big[ \wedge h \Big]^k$$

Frolov, Krtous, Kubiznak, Page '06-'08

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- Ellipsoidal coordinates and "canonical" frames

$$e_t = -\sqrt{\frac{R^2}{FR(R-Mr)}}\left[\partial_t - \sum_k \frac{a_k}{r^2+a_k^2}\partial_{\phi_k}\right], \quad e_r = \sqrt{\frac{R-Mr}{FR}}\partial_r,$$

$$e_i = -\sqrt{\frac{H_i}{d_i(r^2+x_i^2)}}\left[\partial_t - \sum_k \frac{a_k}{a_k^2-x_i^2}\partial_{\phi_k}\right], \quad e_{x_i} = \sqrt{\frac{H_i}{d_i(r^2+x_i^2)}}\partial_{x_i}$$

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  - separation constants: coefficients of one polynomial $P_{n-1}$
- The ODEs should have counterparts for fields with higher spins

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$$l_\pm^\mu\partial_\mu = \frac{R}{\sqrt{\Delta}}\left\{\frac{\Delta}{R}\partial_r \pm \left[\partial_t - \sum_k \frac{a_k}{r^2+a_k^2}\partial_{\phi_k}\right]\right\}, \quad \Delta = R - Mr,$$

$$\left[m_\pm^{(j)}\right]^\mu \partial_\mu = \sqrt{H_j}\left\{\partial_{x_j} \pm i\left[\partial_t - \sum_k \frac{a_k}{a_k^2-x_j^2}\partial_{\phi_k}\right]\right\}$$

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  - recall the frames
  - ... and combine them

$$l_\pm^\mu \partial_\mu = \frac{R}{\sqrt{\Delta}} \left\{ \frac{\Delta}{R} \partial_r \pm \left[ \partial_t - \sum_k \frac{a_k}{r^2 + a_k^2} \partial_{\phi_k} \right] \right\}, \quad \Delta = R - Mr,$$

$$\left[ m_\pm^{(j)} \right]^\mu \partial_\mu = \sqrt{H_j} \left\{ \partial_{x_j} \pm i \left[ \partial_t - \sum_k \frac{a_k}{a_k^2 - x_j^2} \partial_{\phi_k} \right] \right\}$$

  - impose an ansatz inspired by 4D

$$l_\pm^\mu A_\mu = \pm \frac{1}{r \pm i\mu} \hat{l}_\pm \Psi, \quad [m_\pm^{(j)}]^\mu A_\mu = \mp \frac{i}{x_j \pm \mu} \hat{m}_\pm^{(j)} \Psi \,,$$

# Maxwell's equations in the Myers–Perry geometry

OL '17

- Ansatz for the gauge field

$$l_\pm^\mu A_\mu = \pm \frac{1}{r \pm i\mu} \hat{l}_\pm \Psi, \quad [m_\pm^{(j)}]^\mu A_\mu = \mp \frac{i}{x_j \pm \mu} \hat{m}_\pm^{(j)} \Psi \,,$$

- Separation of variables and the "master equations"

$$D_j \frac{d}{dx}\left[\frac{H_j}{D_j} X_j'\right] + \left\{\frac{2\Lambda}{D_j} - H_j W_j^2 - \Lambda + P_{n-2}[-x_j^2] D_j\right\} X_j = 0,$$

$$D_r \frac{d}{dr}\left[\frac{\Delta}{D_r}\dot{\Phi}\right] - \left\{\frac{2\Lambda}{D_r} - \frac{R^2 W_r^2}{\Delta} - \Lambda + P_{n-2}[r^2] D_r\right\} \Phi = 0,$$

$$\Omega = \omega - \sum \frac{m_i a_i}{\Lambda_i}, \quad W_j = \omega - \sum \frac{m_k a_k}{a_k^2 - x_j^2}, \quad W_r = \omega - \sum \frac{m_k a_k}{a_k^2 + r^2}.$$

- Equations cover scalar and photon

scalar : $D_r = D_j = 1, \quad \forall \Lambda;$

vector : $\begin{array}{l} D_j = 1 - \frac{x_j^2}{\mu^2} \\ D_r = 1 + \frac{r^2}{\mu^2} \end{array}, \qquad \Lambda = \frac{\Omega}{\mu} \prod \Lambda_k, \quad \Lambda_i = (a_i^2 - \mu^2).$

- Summary: separable solutions for $(D-2)$ polarizations in all $D$

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