Subleading Microstate Counting of AdS_4 Black Hole Entropy

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Motivation

$$S = \frac{k_B c^3}{\hbar} \frac{A}{4G_N}$$

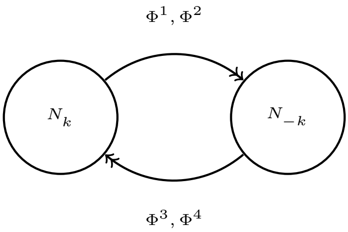
- A confluence of thermodynamical, relativistic, gravitational, and quantum aspects. Hydrogen atom of QG. [Strominger-Vafa].
- An explicit example in $\text{AdS}_4/\text{CFT}_3$: The large-$N$ limit of the topologically twisted index of ABJM correctly reproduces the leading term in the entropy of magnetically charged black holes in asymptotically AdS_4 spacetimes [Benini-Hristov-Zaffaroni].
- Extended also to: dyonic black holes, black holes with hyperbolic horizons and black holes in massive IIA theory.
- Agreement has been shown beyond the large N limit by matching the coefficient of $\log N$ [Liu-PZ-Rathee-Zhao] (Beyond Bohr energies).

Outline

- The Topological Twisted Index of ABJM Theory beyond large N (logarithmic corrections).
- Magnetically Charged Asymptotically AdS_4 Black Holes.
- Logarithmic Corrections in Quantum Supergravity
- The quantum entropy formula for asymptotically AdS black holes.
- Conclusions

ABJM Theory

- ABJM: A 3d Chern-Simons-matter theory with $U(N)_k \times U(N)_{-k}$ gauge group with opposite integer levels. The matter sector contains four complex scalar fields $\Phi_I, (I = 1, 2, 3, 4)$ in the bifundamental representation $(\mathbf{N}, \bar{\mathbf{N}})$, together with their fermionic partners. The theory is superconformal and has $\mathcal{N} = 6$ supersymmetry generically but for $k = 1, 2$, the symmetry is enhanced to $\mathcal{N} = 8$. The global symmetry that is manifest in the $\mathcal{N} = 2$ notation is $SU(2)_{1,2} \times SU(2)_{3,4} \times U(1)_T \times U(1)_R$.



The Topologically Twisted Index of ABJM Theories

- The topologically twisted index for three dimensional $\mathcal{N}=2$ field theories was defined in [Benini-Zaffaroni] (Honda '15, Closset '15) by evaluating the supersymmetric partition function on $S^1\times S^2$ with a topological twist on S^2.
- Hamiltonian: The supersymmetric partition function of the twisted theory, $Z(n_a,\Delta_a)=\mathrm{Tr}\,(-1)^F e^{-\beta H} e^{iJ_a\Delta_a}$. It depends on the fluxes, n_a, through H and on the chemical potentials Δ_a.
- The topologically twisted index for $\mathcal{N}\geq 2$ supersymmetric theories on $S^2\times S^1$ can be computed via supersymmetric localization.
- The supersymmetric localization computation of the topologically twisted index can be extended to theories defined on $\Sigma_g\times S^1$.

General form of the Index

- Background:

$$ds^2 = R^2(d\theta^2 + \sin^2\theta d\phi^2) + \beta^2 dt^2, \quad A^R = \frac{1}{2}\cos\theta d\phi.$$

- The index can be expressed as a contour integral:

$$Z(n_a, y_a) = \sum_{\mathfrak{m}\in\Gamma_{\mathfrak{h}}} \oint_{\mathcal{C}} Z_{int}(x, \mathfrak{m}; n_a, y_a).$$

- Z_{int} meromorphic form, Cartan-valued complex variables $x = e^{i(A_t + i\beta\sigma)}$, lattice of magnetic gauge fluxes $\Gamma_{\mathfrak{h}}$.
- Flavor magnetic fluxes $\mathfrak{n}$ and fugacities $y_a = e^{i(A_t^a + i\beta\sigma^a)}$.
- Localization: $Z_{int} = Z_{\text{class}} Z_{\text{one-loop}}$.
- E.G.: $Z^{CS}_{class} = x^{km}$, $Z^{gauge}_{1-loop} = \prod_{\alpha\in G}(1 - x^\alpha)(idu)^r$, r – rank of the gauge group, α – roots of G and $u = A_t + i\beta\sigma$.

- The topologically twisted index for ABMJ theory:

$$Z(y_a,n_a) = \prod_{a=1}^{4} y_a^{-\frac{1}{2}N^2 n_a} \sum_{I\in BAE} \frac{1}{\det \mathbb{B}} \times \frac{\prod_{i=1}^N x_i^N \tilde{x}_i^N \prod_{i\neq j}\left(1-\frac{x_i}{x_j}\right)\left(1-\frac{\tilde{x}_i}{\tilde{x}_j}\right)}{\prod_{i,j=1}^N \prod_{a=1,2}(\tilde{x}_j - y_a x_i)^{1-n_a} \prod_{a=3,4}(x_i - y_a \tilde{x}_j)^{1-n_a}}.$$

- Contour integral $\to$ Evaluation (Poles): $e^{iB_i} = e^{i\tilde{B}_i} = 1$

$$e^{iB_i} = x_i^k \prod_{j=1}^N \frac{(1-y_3\frac{\tilde{x}_j}{x_i})(1-y_4\frac{\tilde{x}_j}{x_i})}{(1-y_1^{-1}\frac{\tilde{x}_j}{x_i})(1-y_2^{-1}\frac{\tilde{x}_j}{x_i})},$$

$$e^{i\tilde{B}_j} = \tilde{x}_j^k \prod_{i=1}^N \frac{(1-y_3\frac{\tilde{x}_j}{x_i})(1-y_4\frac{\tilde{x}_j}{x_i})}{(1-y_1^{-1}\frac{\tilde{x}_j}{x_i})(1-y_2^{-1}\frac{\tilde{x}_j}{x_i})}.$$

- The $2N\times 2N$ matrix $\mathbb{B}$ is the Jacobian relating the $\{x_i, \tilde{x}_j\}$ variables to the $\{e^{iB_i}, e^{i\tilde{B}_j}\}$ variables

Algorithmic Summary

- Recall, the chemical potentials Δ_a according to $y_a = e^{i\Delta_a}$, and change of variables $x_i = e^{iu_i}$, $\tilde{x}_j = e^{i\tilde{u}_j}$.

$$0 = ku_i - i\sum_{j=1}^{N}\left[\sum_{a=3,4}\log\left(1 - e^{i(\tilde{u}_j - u_i + \Delta_a)}\right) - \sum_{a=1,2}\log\left(1 - e^{i(\tilde{u}_j - u_i - \Delta_a)}\right)\right] - 2\pi n_i,$$
$$0 = k\tilde{u}_j - i\sum_{i=1}^{N}\left[\sum_{a=3,4}\log\left(1 - e^{i(\tilde{u}_j - u_i + \Delta_a)}\right) - \sum_{a=1,2}\log\left(1 - e^{i(\tilde{u}_j - u_i - \Delta_a)}\right)\right] - 2\pi \tilde{n}_j.$$

- The topologically twisted index: (i) solve these equations for $\{u_i, \tilde{u}_j\}$; (ii) insert the solutions into the expression for Z.

The large-N limit

- In the large-N limit, the eigenvalue distribution becomes continuous, and the set $\{t_i\}$ may be described by an eigenvalue density $\rho(t)$.

$$u_i = iN^{1/2}\,t_i + \pi - \tfrac{1}{2}\delta v(t_i), \qquad \tilde{u}_i = iN^{1/2}\,t_i + \pi + \tfrac{1}{2}\delta v(t_i),$$

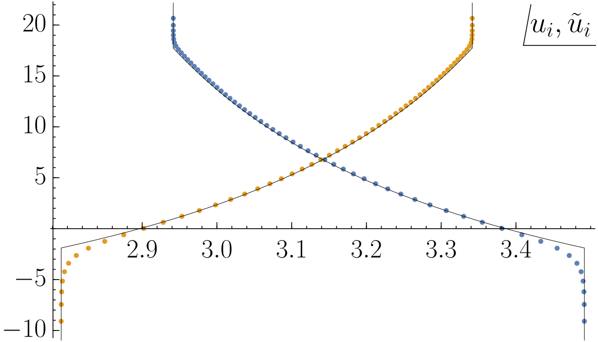


Figure: Eigenvalues for $\Delta_a = \{0.4, 0.5, 0.7, 2\pi - 1.6\}$ and $N = 60$.

- Description of the eigenvalue distribution.

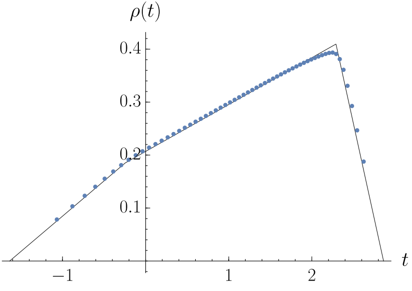


Figure: The eigenvalue density $\rho(t)$ and the function $\delta v(t)$ for $\Delta_a = \{0.4, 0.5, 0.7, 2\pi - 1.6\}$ and $N = 60$, compared with the leading order expression.

$$\mathrm{Re}\log Z = -\frac{N^{3/2}}{3}\sqrt{2\Delta_1\Delta_2\Delta_3\Delta_4}\sum_a \frac{n_a}{\Delta_a}$$

Beyond Large N: Numerical Fits

Δ_1	Δ_2	Δ_3	f_1	f_2	f_3
$\pi/2$	$\pi/2$	$\pi/2$	3.0545	-0.4999	-3.0466
$\pi/4$	$\pi/2$	$\pi/4$	$4.2215 - 0.0491n_1$ $-0.1473n_2 - 0.0491n_3$	$-0.4996 + 0.0000n_1$ $+0.0000n_2 + 0.0000n_3$	$-4.1710 - 0.2943n_1$ $+0.0645n_2 - 0.2943n_3$
0.3	0.4	0.5	$7.9855 - 0.2597n_1$ $-0.5833n_2 - 0.6411n_3$	$-0.4994 - 0.0061n_1$ $-0.0020n_2 - 0.0007n_3$	$-9.8404 - 0.9312n_1$ $-0.0293n_2 + 0.3739n_3$
0.4	0.5	0.7	$6.6696 - 0.1904n_1$ $-0.4166n_2 - 0.4915n_3$	$-0.4986 - 0.0016n_1$ $-0.0008n_2 - 0.0001n_3$	$-7.5313 - 0.6893n_1$ $-0.1581n_2 + 0.2767n_3$

- Numerical fit for:

$$\mathrm{Re}\log Z = \mathrm{Re}\log Z_0 + f_1 N^{1/2} + f_2 \log N + f_3 + \cdots$$

- The values of N used in the fit range from 50 to $N_{\max}$ where $N_{\max} = 290, 150, 190, 120$ for the four cases, respectively.
- The index is independent of the magnetic fluxes in the special case $\Delta_a = \{\pi/2, \pi/2, \pi/2, \pi/2\}$

- In the large-N limit, the $k = 1$ index takes the form

$$F = -\frac{N^{3/2}}{3}\sqrt{2\Delta_1\Delta_2\Delta_3\Delta_4}\sum_a \frac{n_a}{\Delta_a} + N^{1/2} f_1(\Delta_a, n_a)$$
$$-\frac{1}{2}\log N + f_3(\Delta_a, n_a) + \mathcal{O}(N^{-1/2}),$$

  where $F = \text{Re} \log Z$.

- The leading $\mathcal{O}(N^{3/2})$ term [BHZ], and exactly reproduces the Bekenstein-Hawking entropy of a family of extremal AdS_4 magnetic black holes admitting an explicit embedding into 11d supergravity, once extremized with respect to the flavor and R-symmetries.
- The $-\frac{1}{2}\log N$ term [Liu-PZ-Rathee-Zhao].

Topologically twisted index on Riemann surfaces

- The topologically twisted index can be defined on Riemann surfaces with arbitrary genus. There is a simple relation between the index on $\Sigma_g \times S^1$ and that on $S^2 \times S^1$: $F_{S^2\times S^1}(n_a, \Delta_a) = (1-g)F_{\Sigma_g\times S^1}(\frac{n_a}{1-g}, \Delta_a)$.
- Since the coefficient of the logarithmic term in $F_{S^2\times S^1}$ does not depend on n_a we simply have

$$F_{\Sigma_g\times S^1}(n_a, \Delta_a) = \cdots - \frac{1-g}{2}\log N + \cdots \,.$$

AdS_4/CFT_3

- Holographically, ABJM describes a stack of N M2-branes probing a $\mathbb{C}^4/\mathbb{Z}_k$ singularity, whose low energy dynamics are effectively described by 11 dimensional supergravity.
- The index is computed for ABJM theory with a topological twist, equivalently, fluxes on S^2. On the gravity side it corresponds to microstate counting of magnetically charged asymptotically AdS$_4$ black holes.

Supergravity solution

- A solution of $\mathcal{N}=2$ gauged sugra with prepotential $F=-2i\sqrt{X^0X^1X^2X^3}$ coming from M theory on $AdS_4\times S^7$ with $U(1)^4\in SO(8)$.
- Background metric :

$$ds^2=-e^{\mathcal{K}(X)}\left(g\,r-\frac{c}{2g\,r}\right)^2dt^2+e^{-\mathcal{K}(X)}\frac{dr^2}{\left(g\,r-\frac{c}{2g\,r}\right)^2}+2e^{-\mathcal{K}(X)}r^2d\Omega_2^2$$

- Magnetic charges

$$F^a_{\theta\phi}=-\frac{n_a}{\sqrt{2}}\sin\theta,\qquad F^1_{tr}=0.$$

Bekenstein-Hawking entropy and Index

- The Bekenstein-Hawking entropy:

$$S(n_a) = \frac{1}{4G_N} A = \frac{2\pi}{G_N} e^{-\mathcal{K}(X_h)} r_h^2 =$$

- Extremize the index $Z(n_a, y_a)$ with respect to y_a coincides with the entropy $\ln \mathrm{Re} Z(n_a, \tilde{y}_a) = S_{BH}$.
- Why extremization? [Cabo-Bizet-Kol-PZ-Papadimitriou-Rathee].
- Goal: Compute one-loop corrections around this sugra background in 11 Sugra.

Logarithmic terms in one-loop effective actions

- One-loop effective action is equivalent to the computations of determinants.
- For a given kinetic operator A one naturally defines the logarithm of its determinant as

$$\frac{1}{2}\,\ln \mathrm{det}' A = \frac{1}{2}\sum_n{}'\;\ln\kappa_n$$

  where prime denotes that the sum is over non-vanishing are eigenvalues, κ_n of A.
- It is further convenient to define the heat Kernerl of the operator A as

$$K(\tau) = e^{-\tau A} = \sum_n e^{-\kappa_n \tau} \mid \phi_n\rangle\langle\phi_n \mid .$$

Logarithmic terms in one-loop effective actions

- The heat kernel contains information about both, the non-zero modes and the zero modes.
- Let n_A^0 be the number of zero modes of the operator A.

$$-\frac{1}{2}\,\ln \det{}' A = \frac{1}{2}\int_{\epsilon}^{\infty}\,\frac{d\tau}{\tau}\,\left(\mathrm{Tr}K(\tau) - n_A^0\right)$$

where ϵ is a UV cutoff.

- At small τ, the Seeley-De Witt expansion for the heat kernel is appropriate:

$$\mathrm{Tr}K(\tau) = \frac{1}{(4\pi)^{d/2}}\sum_{n=0}^{\infty}\tau^{n-d/2}\int d^d x\,\sqrt{g}\,a_n(x,x).$$

Logarithmic terms in one-loop effective actions

- Since, non-zero eigenvalues of a standard Laplace operator A scale as L^{-2}, it is natural to redefine $\bar{\tau} = \tau / L^2$.

$$-\frac{1}{2}\ \ln \det{}' A = \frac{1}{2}\int_{\bar{\epsilon}}^{\infty} \frac{d\bar{\tau}}{\bar{\tau}} \left(\sum_{n=0}^{\infty} \frac{1}{(4\pi)^{d/2}}\ \bar{\tau}^{\,n-d/2}\ L^{2n-d} \int d^d x\ \sqrt{g}\, a_n(x,x) - n_A^0 \right).$$

- The logarithmic contribution to $\ln \det{}' A$ comes from the term $n = d/2$,

$$-\frac{1}{2}\ \ln \det{}' A = \left(\frac{1}{(4\pi)^{d/2}} \int d^d x\ \sqrt{g}\, a_{d/2}(x,x) - n_A^0 \right) \log L + \ldots .$$

- On very general grounds (diffeomorphism), the coefficient $a_{d/2}$ vanishes in odd-dimensional spacetimes,.

Quantum Supergravity: Key Facts

- The coefficient of the logarithmic term is well-defined.
- In odd-dimensional spaces the coefficient of the $\log$ can only come from zero modes or boundary modes.
- Corrections to entropy from one-loop part of the partition function:

$$S_1 = \lim_{\beta\to\infty} (1 - \beta\partial_\beta) \left(\sum_D (-1)^D (\tfrac{1}{2}\log \mathrm{det}' D) + \Delta F_0 \right),$$

- D stands for kinetic operators corresponding to various fluctuating fields and $(-1)^D = -1$ for bosons and 1 for fermions.
- The zero modes are accounted for separately by

$$\Delta F_0 = \log \int [d\phi]|_{D\phi=0},$$

where $\exp(-\int d^d x \sqrt{g} \phi D \phi) = 1$.

Quantum Supergravity: Key Facts

- The structure of the logarithmic term in 11d Sugra:

$$\log Z[\beta, \dots] = \sum_{\{D\}} (-1)^D (\beta_D - 1) n_D^0 \log L + \Delta F_{\text{Ghost}}.$$

- Subtract the zero modes (-1) and add them appropriately due to integration over zero modes (β_D).
- The ghost contributions are treated separately.

Zero modes and L^2 cohomology

- For zero modes of A_p in compact space, one requires $\langle dA_p, dA_p \rangle = 0$. This amounts to requiring A_p to be closed. But A_p and $A_p + d\alpha_{p-1}$ are gauge equivalent, and the redundant contributions in the path integral are canceled by the Faddeev-Popov procedure. The number of the zero modes is the dimension of the p-th de-Rham cohomology.
- For a non-normalizable $p-1$ form, the gauge transformation $d\alpha_{p-1}$ can be normalizable and included in the physical spectrum, yet Faddeev-Popov procedure can only cancel gauge transformations with normalizable α_{p-1}.
- A physical spectrum with some pure gauge modes with non-normalizable gauge parameter is ubiquitous in one-loop gravity computations in AdS [Sen].
- Mathematically, one considers L^2 cohomology, $H^p_{L^2}(M, \mathbb{R})$.

Zero modes

- The Euler characteristics contains relevant information about the number of zero modes

$$\chi(M) = \sum_p (-1)^p \mathrm{dim}\mathcal{H}^p(M, \mathbb{R}).$$

- There is an appropriate modification of the Gauss-Bonnet theorem $\int^{\mathrm{Reg}} \mathrm{Pf}(R)$ in the presence of a boundary:

$$\chi = \frac{1}{32\pi^2}\left(\int E_4 - 2 \int \epsilon_{abcd}\, \theta^a_b\, \mathcal{R}^c_d + \frac{4}{3} \int \epsilon_{abcd}\, \theta^a_b\, \theta^c_e\, \theta^e_d \right)$$
$$= 2.$$

- Euler density: $E_4 = \frac{1}{64}\left(R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right)$.
- Generalize to blackh hole with horizon of Σ_g: $\chi = 2(1-g)$.

Zero modes

- In the non-extremal case the topology of the black hole is homotopic to its horizon Σ_g due to the contractible (t, r) directions.
- The Euler characteristic of the non-extremal black hole is simply $\chi_{\text{BH}} = 2(1-g)$.
- It also indicates that all but the second relative de-Rham cohomology vanish

$$\text{dim}^R\mathcal{H}^2_{L^2}(M,\mathbb{R}) = \int^{\text{Reg}} \text{Pf}(R) = \chi_{\text{BH}} = 2(1-g).$$

- The no- extremal black hole background has only two-form zero modes and their regularized number is:

$$n_2^0 = 2(1-g).$$

- Where are the 2-forms in 11d Sugra?

Quantizing a p-form

- The general action for quantizing a p-form A_p requires a set of $(p-j+1)$-form ghost fields, with $j = 2, 3, \ldots, p+1$.
- The ghost is Grassmann even if j is odd and Grassmann odd if j is even [Siegel '80]

$$\Delta F_{\text{Ghost}} = \sum_j (-1)^j (\beta_{A_{p-j}} - j - 1) n^0_{A_{p-j}} \log L.$$

Quantizing C_3 and zero modes

- The quantization of the three-form $C_{\mu\nu\rho}$ introduces 2 two-form ghosts that are Grassmann odd, 3 one-form ghosts that are Grassmann even and 4 scalar ghosts that are Grassmann odd[Siegel '80].
- Note that $C_{\mu\nu\rho}$ itself can't decompose as a massless two-form in the black hole background and a massless one-form in the compact dimension since S^7 does not admit any non-trivial 1-cycles.
- The only two-form comes from the two-form ghosts when quantizing $C_{\mu\nu\rho}$

Quantizing C_3 and zero modes

- Ghost contribution to one-loop effective action:

$$\Delta F = \Delta F_{\text{Ghost2form}}.$$

- The 2-form ghost A_2 in 11d has action

$$S_2 = \int A_2 \wedge \star(\delta d + d\delta)^2 A_2,$$

- The logarithmic term in the one-loop contribution to the entropy is

$$(2 - \beta_2) n_2^0 \log L,$$

- Recall β_2 comes from integrating the zero modes in the path integral, and the minus sign takes care of the Grassmann odd nature of A_2.

Zero modes: Two form zero modes

- The properly normalized measure is $\int d[A_{\mu\nu}]\exp(-L^7\int d^{11}x\sqrt{g^{(0)}}g^{(0)\mu\nu}g^{(0)\rho\sigma}A_{\mu\rho}A_{\nu\sigma})=1$, where we single out the L dependence of the metric, $g^{(0)}_{\mu\nu}=\frac{1}{L^2}g_{\mu\nu}$. Thus the normalized measure is $\prod_x d(L^{\frac{7}{2}}A_{\mu\nu})$. For each zero mode, there is a $L^{\frac{7}{2}}$ factor. Thus in the logarithmic determinant, one has $\beta_2=\frac{7}{2}$.
- The $\log L$ contribution to the thermal entropy in the extremal background is:

$$\log Z[\beta,\dots]=-3(1-g)\log L+\cdots\,.$$

Final Result

- The coefficient of the logarithmic term does not depend on β

$$S_{\text{1-loop}} = (1-\beta\partial_\beta)(-3(1-g)\log L) + \cdots$$
$$= -3(1-g)\log L + \cdots .$$

- As this is β independent, it is also valid in the extremal limit, $\beta \to \infty$.
- The AdS/CFT dictionary establishes that $L \sim N^{1/6}$ leading to a logarithmic correction to the extremal black hole entropy of the form

$$S = \cdots - \frac{1-g}{2}\log N + \cdots ,$$

- Perfectly agrees with the microscopic result!!!

Entropy Formula in AdS

- The entropy of asymptotically AdS black holes can be computed from the near horizon geometry using the Entropy function and attractors [Morales-Samtleben '06, Goulart '15]
- A clean discussion of the holographic computation (justifying the full action) is presented in [Cabo-Bizet-Kol-LPZ-Papadimitriou-Rathee].
- The Quantum Entropy Formula has been successful in asymptotically flat black holes [Sen].

A puzzle: Quantum Entropy Formula

- The near horizon geometry: $AdS_2 \times M_9$, M_9 is a S^2 bundle over S^7 with $\{n_a\}$.
- The fluctuation of the metric to the lowest order can be summarized as

$$h_{\mu\nu}(x,y) = \begin{cases} h_{\alpha\beta}(x)\phi(y), \\ h_{\alpha i} = \sum_a A^a_\alpha(x) K^a_i(y), \\ \phi(x) h_{ij}(y), \end{cases}$$

where we use (x^α, y^i) to denote AdS$_2$ and M_9 coordinates, respectively, and $K^{ai}(y)\partial_i$ is a killing vector of M_9.

The near horizon geometry: $AdS_2 \times M_9$

- Metric: S^2 coordinates on by (θ, ϕ), X_i's are constant with $\prod X_i = 1$, $\Delta = \sum_{i=1}^4 X_i \mu_i^2$, and $\sum_{i=1}^4 \mu_i^2 = 1$:

$$ds_9^2 = \Delta^{\frac{2}{3}} ds_{S^2}^2 + \frac{4}{\Delta^{\frac{1}{3}}} \sum_{i=1}^{4} \frac{1}{X_i} \left(d\mu_i^2 + \mu_i^2 (d\psi_i + \frac{n_i}{2} \cos\theta d\phi)^2 \right),$$

- The metric admits seven Killing vectors ($SU(2)$ isometries extend to the whole bundle:

$$\Big\{ \cos\phi \partial_\theta - \cot\theta \sin\phi \partial_\phi + \sum_j \frac{n_j}{2} \frac{\sin\phi}{\sin\theta} \partial_{\psi_j},$$

$$- \sin\phi \partial_\theta - \cot\theta \cos\phi \partial_\phi + \sum_j \frac{n_j}{2} \frac{\cos\phi}{\sin\theta} \partial_{\psi_j}, \ \partial_\phi \Big\},$$

$$\Big\{ \partial_{\psi_i} \Big\},$$

where $i = 1, 2, 3, 4$, and the Killing vectors span the algebra of the isometry group $SU(2) \times U(1)^4$.

Metric Contribution

- The graviton zero modes therefore contribute in two ways: a graviton in AdS_2, and gauge fields corresponding to Killing vectors of M_9.
- The logarithmic correction due to the 11-dimensional graviton is given by

$$\begin{aligned} \Delta F_h &= (\beta_h - 1)(1 \times n_g^0 + 7 \times n_A^0) \log L \\ &= \left(\frac{11}{2} - 1\right) [1 \times (-3) + 7 \times (-1)] \log L \\ &= -45 \log L. \end{aligned}$$

Quantum Entropy Formula

- Finally, adding all the contributions: graviton, gravitino, 3-form and ghost leads to the total logarithmic correction ($N \sim L^6$):

$$\Delta F = \left(-45 + 36 - \frac{3}{2} - \frac{3}{2}\right) \log L = -12 \log L \sim -2 \log N.$$

- The same answer was simultaneously obtained by (Jeon-Lal '17).
- This result does not match the logarithmic term of the topologically twisted index which has a coefficient $-1/2$.
- The quantum entropy formula counts near horizon degrees of freedom, in AdS it requires a revision.
- Hair degrees of freedom in AdS (HHH superconductor).

Conclusions

- A particular thermal-based limit to the extremal black hole, agreeing with some observations of [Sen].
- The degrees of freedom do not live locally at the horizon. Corrections to the Quantum Entropy Formula, extra hair in AdS [Sen].
- 't Hooft limit:$\lambda = N/k$ held fixed as $N \to \infty$? There are already problems for the free energy of $AdS_4 \times \mathbb{CP}^3$ - an even-dimensional spacetime, the full computation with no simplifications.
- Cetain black strings in AdS_5 and the $\mathcal{N} = 4$ SYM index on $T^2 \times S^2$.
- A *precise setup* to attack important questions of black hole physics in the AdS/CFT.