The topologically twisted index of $\mathcal{N} = 4$ super-Yang-Mills on $T^2 \times S^2$ and the elliptic genus

Junho Hong and James T. Liu

University of Michigan

Great Lakes Strings 2018

[arXiv:1804.04592]

What had been studied

Precision holography

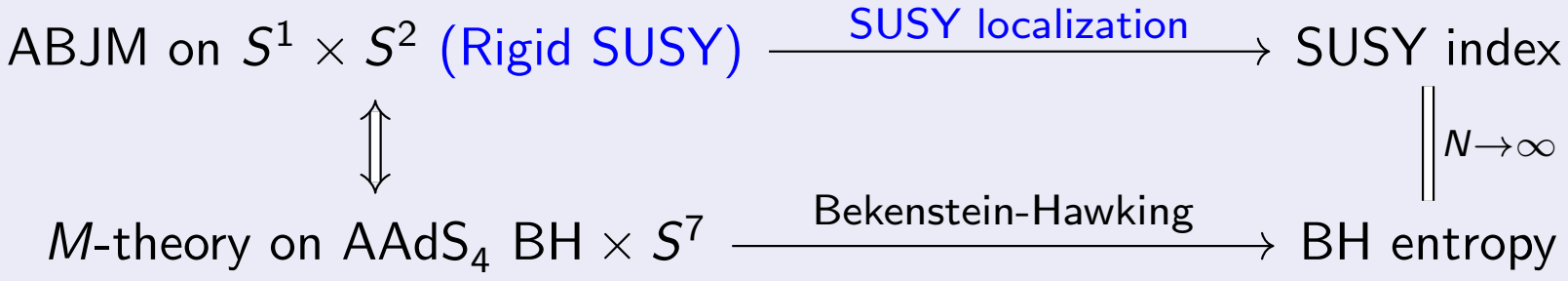
Tries to make AdS/CFT more solid by comparing dual physical quantities

Two recent techniques:

1. SUSY localization
   - Computes "exact" CFT results (e.g. SUSY index)
2. Rigid SUSY
   - Constructs SUSY theories on curved backgrounds (with a topological twist)

What had been studied

3d Example [**Benini 16**]



First counting of $AAdS_4$ BH microstates!

What had been studied

4d Example?

$$\begin{array}{ccc} \mathcal{N}=4\ SYM \text{ on } T^2\times S^2 \text{ (Rigid SUSY)} & \xrightarrow{\text{SUSY localization}} & \text{SUSY index} \\ \Updownarrow & & \Big\| \ N\to\infty? \\ \text{Type IIB SUGRA on AAdS}_5 \text{ BS} \times S^5? & \xrightarrow{\text{Bekenstein-Hawking}} & \text{BS entropy} \end{array}$$

Remaining issues:

- Field theory side: explicit expression for SUSY index and its large N limit?
- Supergravity side: dual AAdS_5 black string SUGRA solution?

We have focused on the issue of the field theory side!

What we have found

SUSY index of $\mathcal{N}=4$ $SU(N)$ SYM on $T^2 \times S^2$ from localization [**Hosseini 17**]

$$Z(\tau;\Delta_a,\mathfrak{n}_a)=\mathcal{A}\sum_{\{u_j\}\in BAEs}\frac{1}{\det\mathbb{B}}\prod_{j\neq k}^{N}\prod_{a=1}^{3}\left(\frac{\theta_1(u_j-u_k;\tau)}{\theta_1(u_j-u_k+\Delta_a;\tau)}\right)^{1-\mathfrak{n}_a}$$

1. Computed the SUSY index
   - Found multiple solutions to the Bethe Ansatz Equations, denoted by $\{m,n,r\}$ $(N=mn,\ r=0,1,\cdots,n-1)$

$$Z(\tau;\Delta_a,\mathfrak{n}_a)=\sum_{n|N}\sum_{r=0}^{n-1}Z_{\{m,n,r\}}(\tau;\Delta_a,\mathfrak{n}_a)$$

   - A weak Jacobi form of weight 0 and indices $-\frac{N^2-1}{2}(1-\mathfrak{n}_a)$

     $\rightarrow$ Elliptic genus of 2d SCFT from KK compactification

What we have found

SUSY index of $\mathcal{N}=4$ $SU(N)$ SYM on $T^2 \times S^2$ with multiple BAE solutions

$$Z(\tau; \Delta_a, \mathfrak{n}_a) = \sum_{n|N} \sum_{r=0}^{n-1} Z_{\{m,n,r\}}(\tau; \Delta_a, \mathfrak{n}_a)$$

2. Conjectured the 'high-temperature' limit ($\beta \to 0^+$, $\tau = i\beta/2\pi$):

$$\log Z|_{\Delta_a=\bar{\Delta}_a} \overset{!}{=} \log Z_{\{1,N,0\}}|_{\Delta_a=\bar{\Delta}_a} = \frac{\pi^2}{6\beta} c_r(\mathfrak{n}_a)$$

at leading $1/\beta$ order.

What we will study

- Field theory side
  - Study the large N limit of the SUSY index
- Supergravity side
  - Construct the dual $AAdS_5$ BS SUGRA solution

Compare $\log Z|_{\Delta_a = \bar{\Delta}_a}$ with $S_{\text{dual BS}}$ in the large N limit

$\Rightarrow$ *Count the microstates of $AAdS_5$ BS using $\mathcal{N} = 4$ SYM!*

 F. Benini, K. Hristov, and A. Zaffaroni, *Black hole microstates in AdS_4 from supersymmetric localization*, *JHEP* **05** (2016) 054 [arXiv:1511.04085].

 S. M. Hosseini, A. Nedelin, and A. Zaffaroni, *The Cardy limit of the topologically twisted index and black strings in AdS_5*, *JHEP* **04** (2017) 014 [arXiv:1611.0937]