The Superconformal Index and the Weyl Anomaly

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Conformal Symmetry and Weyl Invariance

Conformal or **Weyl invariant** theory is unchanged by

$$g_{ab} \to e^{-2\sigma(x)} g_{ab}$$

Invariance means the stress tensor satisfies $T^{\mu}_{\mu} = 0$

This symmetry might be **anomalous**– then $\langle T \rangle = g^{\mu\nu} \langle T_{\mu\nu} \rangle \neq 0$

$\langle T \rangle$ can be given by curvature invariants. In 6D:

$$\langle T \rangle = -aE_6 + (c_1 I_1 + c_2 I_2 + c_3 I_3) + D_\mu J^\mu,$$

Euler density: $E_6 = \epsilon\epsilon RRR$

Contractions of the Weyl tensor: $I_i = C^3$

Holographic computations of the Weyl Anomaly

$$c = \frac{c_2 - c_3}{32}, \qquad c' = \frac{c_1 - 4c_2}{192}, \qquad c'' = \frac{c_1 - 2c_2 + 6c_3}{192}.$$

Holographic computation at leading order Henningson, Skenderis (1998)

In 6D, one-loop corrections are still an active area of research

- δa was computed for all spins using AdS with a sphere boundary Beccaria, Tseytlin (2014)
- We computed $\delta(c - a)$ for spins ≤ 2 on Ricci-flat backgrounds Liu, McPeak (2017)
- With minimal (1,0) SUSY, c'' vanishes
- With (2,0) SUSY, c' and c'' vanish

Superconformal Index

The 6D **superconformal index**:

$$\mathcal{I}(p,q,s) = \mathrm{Tr}_{\mathcal{H}}(-1)^{j_1+j_3} e^{-\beta\delta} q^{\hat{\Delta}} s^{j_1} p^{j_2}$$

.

$\hat{\Delta} = \Delta - \frac{1}{2}k$

$\delta = \hat{\Delta} - \frac{3}{2}k - \frac{1}{2}(j_1 + 2j_2 + 3j_3)$

Compute the index for short multiplets– general structure is:

$$\mathcal{I}(p,q,s) \sim q^{\hat{\Delta}} \frac{\chi_{j_1 j_2}(s,p)}{\mathcal{D}(p,q,s)}$$

χ is the $SU(3)$ character

$\mathcal{D}$ comes from superconformal descendents

Group Theory Invariants

The the anomaly and the index may both be written in terms of $SU(3)$ invariants

$d(j_1, j_2)$ is the dimension

$I(j_1, j_2)$ are the Dynkin indices– e.g. $Tr[T_a^R T_b^R] = 2I_2(R)\delta_{ab}$

For $SU(3)$,

$$d(j_1, j_2) = \tfrac{1}{2}(j_1 + 1)(j_2 + 1)(j_1 + j_2 + 2)$$
$$I_2(j_1, j_2) = \frac{1}{12} d(j_1, j_2)[j_1^2 + 3j_1 + j_1 j_2 + j_2^2 + 3j_2],$$
$$I_3(j_1, j_2) = \frac{1}{60} d(j_1, j_2)(j_1 - j_2)(j_1 + 2j_2 + 3)(2j_1 + j_2 + 3),$$
$$I_{2,2}(j_1, j_2) = \frac{3}{5} I_2(j_1, j_2)\left(8\frac{I_2(j_1, j_2)}{d(j_1, j_2)} - 1\right)$$

Differential Operators

$$\mathcal{I}(p,q,s) \sim q^{\hat{\Delta}} \frac{\chi_{j_1 j_2}(s,p)}{\mathcal{D}(p,q,s)}$$

We may write the Dynkin indices in terms of differential operators acting on the character:

$$\begin{aligned}
d(j_1,j_2) &= \chi_{(j_1,j_2)}(s,p)\Big|_{s=p=1} \\
\hat{I}_2(j_1,j_2) &= \tfrac{1}{2}(s\partial_s)^2 \chi_{(j_1,j_2)}(s,p)\Big|_{s=p=1} \\
\hat{I}_3(j_1,j_2) &= (p\partial_p)(s\partial_s)^2 \chi_{(j_1,j_2)}(s,p)\Big|_{s=p=1} \\
\hat{I}_{2,2}(j_1,j_2) &= \tfrac{1}{2}(s\partial_s)^4 \chi_{(j_1,j_2)}(s,p)\Big|_{s=p=1}
\end{aligned}$$

Sum Over Multiplets

Fields in representations $(\Delta, j_1, j_2, j_3)_k$ of $U(1) \times SU(4) \times SU(2)_R$

Short multiplets contribute to δa, long multiplets give 0

For (1,0) theory, we find that $\delta a \sim \mathcal{A}(j_1, j_2, \hat{\Delta})$

$$\mathcal{A}(j_1, j_2, \hat{\Delta}) \sim -10 \left(\frac{4}{3}\hat{\Delta} - 2\right)^4 d(j_1, j_2)$$
$$+ 20 \left(\frac{4}{3}\hat{\Delta} - 2\right)^2 [4 I_2(j_1, j_2) + d(j_1, j_2)] + \frac{530}{9} \left(\frac{4}{3}\hat{\Delta} - 2\right) I_3(j_1, j_2)$$
$$- \frac{80}{9}[I_{2,2}(j_1, j_2) + 3 I_2(j_1, j_2)] - \frac{11}{3} d(j_1, j_2),$$

$\mathcal{A}$ depends on $(\hat{\Delta}, j_1, j_2)$ of lowest representation in the multiplet

Results

We find the operator for δa,

$$\delta a = \mathcal{O}_a \, \mathcal{D}(p,q,s) \mathcal{I}(p,q,s) \Big|_{p=q=s=1}$$

where

$$\mathcal{O}_a = \frac{1}{2^5 \cdot 6!} \Bigg[-10 \left(\frac{4}{3} q \partial_q - 2 \right)^4 + 20 \left(\frac{4}{3} q \partial_q - 2 \right)^2 (4 \hat{I}_2 + 1) + \frac{530}{9} \left(\frac{4}{3} q \partial_q - 2 \right) \hat{I}_3 - \frac{80}{9} (\hat{I}_{2,2} + 3 \hat{I}_2) - \frac{11}{3} \Bigg]$$

Operator for $(c - a)$

We only know $\delta(c-a)$ values for shortened multiplets starting at spin $(0,0,0)$

However $\delta(c-a)$ is much simpler than a, which lets us *almost* determine the operator:

$$\mathcal{O}_{(c-a)} = \frac{1}{2^5 \cdot 6!}\left[-90\left(\frac{4}{3}q\partial_q - 2\right)\hat{I}_3 + 1 + \lambda(\hat{I}_{2,2} - \hat{I}_2)\right]$$

Summary

- Superconformal index and Weyl anomaly both non-zero only for short multiplets
- Both can be written in terms of $SU(3)$ invariants
- We find an operator which returns δa when acting on the index
- $\delta(c - a)$ operator is determined up to one coefficient
- Fixing it requires anomaly for multiplets with spin> 2

Thank you!