Are Galileon Theories Supersymmetrizable?

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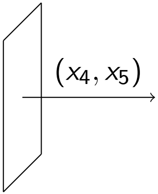
Outline

- Goal: Classify and understand properties of low-energy EFTs arising from spontaneous symmetry breaking
- Method: Use on-shell tree-level amplitudes and their soft behaviour

Brane Teaser

- A 3-brane in a 6-d space
- Transverse directions x_4, x_5, longitudinal directions $x^\mu = (x_0, x_1, x_2, x_3)$
- Embedding of the brane is described by two real scalar fields $\Rightarrow$ one complex scalar field ϕ



Spontaneously Broken Symmetries of ϕ

6-d Poincaré group is spontaneously broken $\Rightarrow$ two Goldstone bosons ϕ, $\bar{\phi}$

- Translations P_4 and P_5, e.g. $\phi \to \phi + a\,\partial_{x_4}\phi$
  $\Rightarrow$ Non-linearly realized as a shift symmetry $\phi \to \phi + c$
- Lorentz rotations $M_{\mu 4}$ and $M_{\mu 5}$
  $\Rightarrow$ Non-linearly realized as a spacetime-dependent shift symmetry $\phi \to \phi + c + v^{\mu} x_{\mu}$

Not obvious from the Lagrangian.
At leading order, only the brane tension term contributes

$$\mathcal{L}_{DBI} = \sqrt{-\det h_{\mu\nu}} = \sqrt{\det\left(\eta_{\mu\nu} - \partial_\mu\phi\ \partial_\nu\bar{\phi}\right)}$$

where $h_{\mu\nu}$ is the induced metric on the brane.

DBI and Galileon

Schematically,

$$\mathcal{L}_{DBI} = \int d^4x \left(\frac{1}{2} \partial^2 \phi \ \bar{\phi} + g_4 \ \partial^4 \phi^2 \bar{\phi}^2 + g_6 \ \partial^6 \phi^3 \bar{\phi}^3 + \cdots \right)$$

- Shift symmetry is 'trivial'
- Space-dependent shift symmetry is 'non-trivial'

Galileon theories

- Sub-leading terms to brane action
- Second order EOM
- Decoupling limit

$$\mathcal{L}_3 = \partial^4 \phi^2 \bar{\phi} + \text{h.c.}$$
$$\mathcal{L}_4 = \partial^6 \phi^2 \bar{\phi}^2$$
$$\mathcal{L}_5 = \partial^8 \phi^3 \bar{\phi}^2 + \text{h.c.}$$

Soft Behaviour

Symmetry	Soft behaviour
$\phi \to \phi + c$	$\mathcal{A}_n(p_\phi) \sim p_\phi$ as $p_\phi \to 0$.
$\phi \to \phi + c + v^\mu x_\mu$	$\mathcal{A}_n(p_\phi) \sim p_\phi^2$ as $p_\phi \to 0$.

SSB of supersymmetry $\Rightarrow$ Goldstino ψ

Non-linear realization of broken SUSY is a shift $\psi \to \psi + \eta$, i.e.

$$\mathcal{A}_n(p_\psi) \sim p_\psi \quad \text{as} \quad p_\psi \to 0$$

Aim: Soft behaviour of amplitudes $\to$ Existence of SUSY Galileons

Supersymmetry

$\mathcal{N} = 1$ DBI has

- Soft behaviour p_ϕ^2 for ϕ, $\bar{\phi}$
- Soft behaviour p_ψ for partner Goldstino
  $\Rightarrow$ Must break $\mathcal{N} = 2$ SUSY to $\mathcal{N} = 1$

What about the Galileon?

- Start with $\mathcal{L}_{\text{Gal}} = g_3 \mathcal{L}_3 + g_4 \mathcal{L}_4 + g_5 \mathcal{L}_5$
- Field redefinition $\phi \to \phi + a(\partial\phi)^2$
- Choose a so that $\tilde{g}_3 = 0$, i.e. $\mathcal{L}_{\text{Gal}} = \tilde{g}_4 \mathcal{L}_4 + \tilde{g}_5 \mathcal{L}_5$
- $\mathcal{N} = 1$ SUSY $\mathcal{L}_4$ in the next talk!

We answer the question of supersymmetrization of $\mathcal{L}_3$ via $\mathcal{L}_4$ and $\mathcal{L}_5$.

Supersymmetrizing the Quintic Galileon $\mathcal{L}_5$

- Find all structures for $\mathcal{A}_5(\phi_1, \bar{\phi}_2, \phi_3, \bar{\phi}_4, \phi_5)$
  - ▷ Lorentz invariance ⇒ function of spinor brackets
  - ▷ Locality ⇒ polynomial
  - ▷ Little group ⇒ does not scale
  - ▷ Fermi-Bose symmetry ⇒ symmetric under $(1 \leftrightarrow 3)$, $(2 \leftrightarrow 4)$, $(3 \leftrightarrow 5)$
  - ▷ Mass dimension ⇒ $[\mathcal{A}_5] = -9$
- Impose soft behaviour p_ϕ^2

$$\begin{aligned}\mathcal{A}_5(\phi_1, \bar{\phi}_2, \phi_3, \bar{\phi}_4, \phi_5) =& \tilde{g}_5 \left[s_{13} s_{25} \left(s_{15} s_{23} + s_{12} s_{35} - s_{13} s_{25} \right) \right. \\ & \left. + (1 \leftrightarrow 5) + (3 \leftrightarrow 5) \right] + (2 \leftrightarrow 4)\end{aligned}$$

Supersymmetrizing the Quintic Galileon $\mathcal{L}_5$

Impose the SUSY Ward identities

$$\mathcal{A}_5(\phi_1, \bar{\phi}_2, \phi_3, \bar{\psi}_4, \psi_5) = \frac{[25]}{[24]} \mathcal{A}_5(\phi_1, \bar{\phi}_2, \phi_3, \bar{\phi}_4, \phi_5)$$

Has a pole as $[24] \to 0$! $\Rightarrow$ not local

There exists no $\mathcal{N} = 1$ SUSY $\mathcal{L}_5$ with p_ϕ^2 soft behaviour
$\Rightarrow$ there is no supersymmetrization of $g_3\mathcal{L}_3$

A SUSY quintic Galileon

- What about a theory having p_ϕ^2 behaviour for the real part of the complex scalar ϕ, but p_ϕ behaviour for the imaginary part?
- We find a consistent solution.

- Does such a theory make sense?
- p_ϕ^2 scalar is a Goldstone boson from 5d Poincaré group breaking
- p_ϕ scalar is an R-axion

Summary

- Galileons arise as terms subleading to DBI in brane actions
- We address the question of possible supersymmetrizations of Galileon theories via on-shell amplitudes
- We make definite statements of non-supersymmetrizability of $\mathcal{L}_3$ and $\mathcal{L}_5$
- We find evidence that supports the existence of a SUSY $\mathcal{L}_5$ with an R-axion

Thank you!