A New Perspective on Nonlinear Supersymmetry

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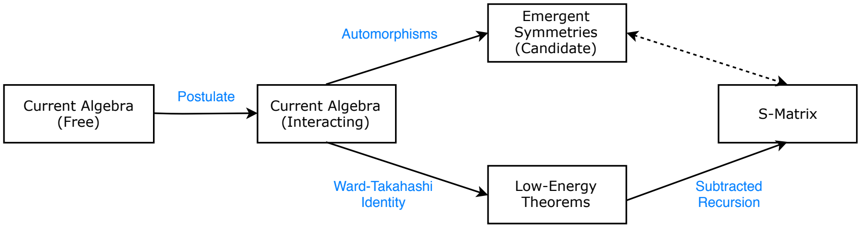
Background

- Field theories for which the massless spectrum consists only of Goldstone modes generically display a surprisingly rich assortment of *emergent* or *accidental* symmetries at very low energies, often with dramatic consequences.
- **Example:** It was only very recently observed that the leading order D3-brane action in 10d Minkowski space ($\mathcal{N}=4$ DBI) has an additional $U(1)_R$ symmetry corresponding to *helicity conservation* [Heydeman, Schwarz, Wen: 1710.02170].
- From the point of view of the low-energy effective action the appearance of such symmetries is a miracle often requiring explanation in UV physics or more exotic ideas (field theory KLT relations).
- **This talk:** Give a new perspective on models with *both* linear and nonlinear supersymmetry using a combination of current algebra and on-shell recursion. From our new point of view such emergent symmetries will be natural and manifest.



Nonlinear Supersymmetry Current Algebra (I)

- We will consider a model which reduces to a free $\mathcal{N} = 1$ vector multiplet

$$S \to S^{(0)} = \int \mathsf{d}^4 x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi \right]$$

- In addition to the usual linear supersymmetry this model also has a second *nonlinear* supersymmetry

$$\delta_\eta^{(2)} \psi_\alpha(x) = \Lambda^2 \eta_\alpha$$

- Generates a current algebra

$$\begin{aligned}
\{\mathcal{Q}_{1\alpha}, \mathcal{S}^{\dagger\mu}_{1\dot{\alpha}}(x)\} &= \sigma_{\nu\alpha\dot{\alpha}} T^{\mu\nu}(x) \\
\{\mathcal{Q}_{2\alpha}, \mathcal{S}^{\dagger\mu}_{2\dot{\alpha}}(x)\} &= \Lambda^4 \sigma^\mu_{\alpha\dot{\alpha}} \\
\{\mathcal{Q}_{1\alpha}, \mathcal{S}^{\dagger\mu}_{2\dot{\alpha}}(x)\} &= \sigma_{\nu\alpha\dot{\alpha}} K^{\mu\nu}(x) \\
[\mathcal{Q}_{2\alpha}, K^{\mu\nu}(x)] &= 0
\end{aligned}$$

- The presence of a constant central term prevents us from constructing an algebra of charges for nonlinear supersymmetries in *infinite volume* [Hughes, Polchinski *Nucl.Phys. B278 (1986)*].
- We **postulate** the existence of an interacting field theory with an isomorphic current algebra and use on-shell recursion to construct its S-matrix.

Nonlinear Supersymmetry Current Algebra (II)

- All of the particles in this model are interpolated by current operators

$$\mathcal{S}^{\mu}_{2\alpha}(x) = \Lambda^2 \sigma^{\mu}_{\alpha\dot{\alpha}} \psi^{\dagger\dot{\alpha}}(x) + ... \qquad K^{\mu\nu}(x) = \Lambda^2 F^{\mu\nu}_{+}(x) + ...$$

- 2-form current $K^{\mu\nu}$ corresponds to *large* gauge transformations of the vector boson

$$\left[\delta^{(2)}_{\eta}, \delta^{(1)}_{\xi^\dagger}\right] A_\mu(x) = -i\eta\sigma_\mu\xi^\dagger = \partial_\mu \left((-i\eta\sigma_\nu\xi^\dagger)x^\nu\right)$$

- Symmetries which act linearly on the physical states must correspond to linear automorphisms of the current algebra

$$\{\mathcal{Q}_{1\alpha}, \mathcal{S}^{\dagger\mu}_{2\dot{\alpha}}(x)\} = \sigma_{\nu\alpha\dot{\alpha}} K^{\mu\nu}(x)$$

(candidate) linear R-symmetries:

$$\mathcal{Q}_{1\alpha} \to e^{i\theta}\mathcal{Q}_{1\alpha} \;\text{ and }\; \mathcal{S}^{\dagger\mu}_{2\dot{\alpha}}(x) \to e^{-i\theta}\mathcal{S}^{\dagger\mu}_{2\dot{\alpha}}(x) \;\text{ or }\; K^{\mu\nu}(x) \to e^{i\theta}K^{\mu\nu}(x)$$

- Current algebra is therefore *compatible* with the conservation of independent chiral charges for both the fermion and boson.

Nonlinear Supersymmetry Current Algebra (III)

- Low-energy theorems for the S-matrix follow from the Ward-Takahashi identities:

$$
\begin{aligned}
&\partial^x_\mu \langle \mathcal{S}^\mu_{2\alpha}(x)\mathcal{S}^{\dagger\mu}_{2\dot\alpha_1}(x_1)K^{\mu_2\nu_2}(x_2)K^{*\mu_3\nu_3}(x_3)\rangle \\
&= i\delta^{(4)}(x-x_1)\langle\{\mathcal{Q}_{2\alpha},\mathcal{S}^{\dagger\mu}_{2\dot\alpha_1}(x_1)\}K^{\mu_2\nu_2}(x_2)K^{*\mu_3\nu_3}(x_3)\rangle \\
&= i\delta^{(4)}(x-x_1)\Lambda^4\sigma^\mu_{\alpha\dot\alpha_1}\langle K^{\mu_2\nu_2}(x_2)K^{*\mu_3\nu_3}(x_3)\rangle
\end{aligned}
$$

- Since there is no operator on the right-hand-side with a (Fourier space) singularity at $p_1^2 = 0$ taking the LSZ projection and soft limit gives a *vanishing* low energy theorem:

$$\lim_{q\to 0}\mathcal{A}_4\left(\psi^-(q)\psi^+(p_1)\gamma^-(p_2)\gamma^+(p_3)\right) = 0.$$

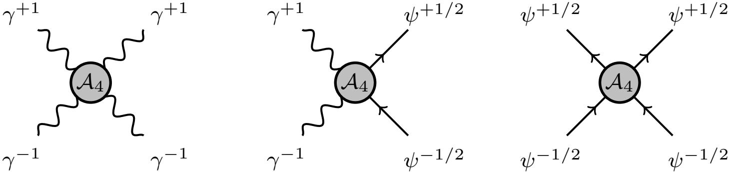
- In general since there are no operators of the form $\{\mathcal{Q}_{2\alpha},\mathcal{O}(x)\}$ which interpolate one-particle states *all S-matrix elements must vanish in the soft Goldstino limit*.

Constructing the S-matrix (I)

- A more careful analysis gives the following low energy theorem:

$$\mathcal{A}_n\left(...\{\epsilon|i\rangle, |i]\}^+_\psi...\right) \sim \epsilon^1, \quad \epsilon \to 0 \quad \Rightarrow \quad \sigma_\psi = 1.$$

- **Idea:** Use the low-energy theorems together with *subtracted on-shell recursion* to construct the tree-level S-matrix [Cheung, Kampf, Novotny, Shen, Trnka : 1611.03137].
- The seed of the recursion is given by constructing the amplitudes of lowest dimension

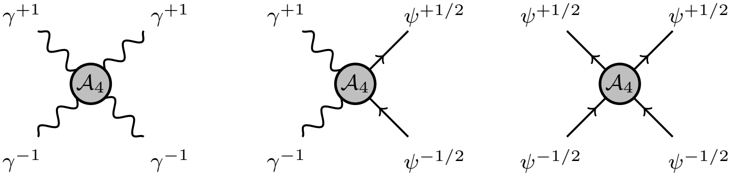


- Explicitly:

$$\mathcal{A}_4\left(1^+_\gamma, 2^-_\gamma, 3^+_\gamma, 4^-_\gamma\right) = \frac{1}{\Lambda^4}[13]^2\langle 24\rangle^2,$$

this choice is *unique* at $\mathcal{O}\left(1/\Lambda^4\right)$ given the assumed linear supersymmetry and low-energy theorems. In particular all other helicity assignments are forbidden.

Constructing the S-matrix (II)



- Constructibility criterion:

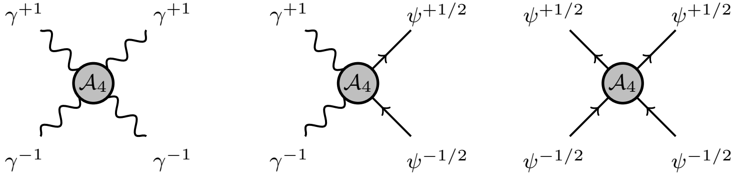
$$4 - n - [g]\left(\frac{n-2}{v-2}\right) - \sum_{i=1}^{n} \sigma_i - \sum_{i=1}^{n} s_i < 0$$

- For the assumed 4-particle amplitudes

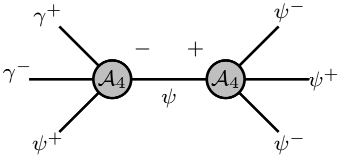
$$4 - n - (-4)\left(\frac{n-2}{4-2}\right) - \frac{1}{2}n_\psi - n_\gamma - n_\psi < 0 \Longrightarrow n_\psi > 0$$

- On-shell supersymmetry Ward identities fix $n_\psi = 0$ amplitudes in terms of the others
- **Result:** The leading contribution to the S-matrix in the deep IR is uniquely generated by subtracted on-shell recursion. At this order the model is identical to the $\mathcal{N} = 1$ supersymmetrization of the Born-Infeld model [Bagger, Galperin: 9608177].

Recursion Implies Conservation

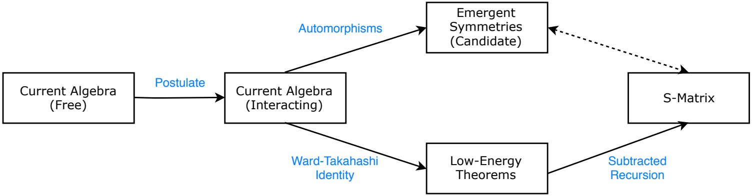


- On-shell recursion is not just an efficient way to construct amplitudes, it places fundamental constraints on the form of the S-matrix.
- **A seemingly trivial observation:** in a recursive model all higher-point amplitudes are given by *gluing* together the fundamental amplitudes:



- **Therefore:** if the fundamental amplitudes conserve an additive quantum number, it must be conserved by *all of the amplitudes!*
- The fundamental 4-point amplitudes manifestly conserve independent chiral charges for both the fermion and vector. All linear automorphism symmetries of the current algebra are symmetries of the S-matrix at low-energies.

Conclusions and Open Problems



- This analysis can be extended almost verbatim to $\mathcal{N} = 4$ DBI and various extended supergravities. In each of these cases the S-matrix is on-shell constructible at leading order and the charge assignments of emergent symmetries can be deduced by inspection of the fundamental amplitudes.
- Beyond leading order in the EFT expansion tree-level contributions from higher dimension operators generally appear at the same order as loop contributions. The Ward-Takahashi identity and consequently the low-energy theorems may be modified by anomalous contributions. The general structure of these contributions and the consequences for the emergent symmetries are *poorly understood* [*c.f.* Bern, Parra-Martinez, Roiban: 1712.03928].
- Fortunately there are a class of protected sub-leading operators which are *IR dominant* over the 1-loop contributions (the Galileon operators) and which may violate the emergent symmetries while remaining compatible with the assumed low-energy theorems. Much more on this point in our forthcoming paper.

Thank you!