An Attractor Mechanism for nAdS$_2$/CFT$_1$ Holography

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$\text{AdS}_2/\text{CFT}_1$ Holography

AdS$_{d+1}$/CFT$_d$ correspondence is confusing for $d = 2$.

- Decoupling limit between worldvolume and bulk geometry fails for $D6$-branes.

- No finite energy excitations possible in AdS$_2$ (or else backreaction spoils asymptotic AdS$_2$).

- Conformal quantum mechanics (CFT$_1$) has no ground state (or other unpleasantries).

$\mathbf{nAdS_2/nCFT_1}$ Holography.

- Variation over $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$ correspondence: holography between ***nearly*** AdS_2 geometry and ***nearly*** CFT_1.

- Conformal symmetry is ***broken spontaneously*** (by boundary conditions) and ***broken explicitly*** (by an anomaly).

- Breaking is "small": cut off AdS_2 before breaking dominates.

- Interesting nCFT_1's realize the symmetry breaking pattern: SYK,....

A New Scale

- The AdS_2 scale ℓ_2 is not a true scale: it is a unit for everything.

- eg. dimensionless scalar masses $m\ell_2$ are essentially the conformal weights

$$h = 1 + \sqrt{1 + m^2\ell_2^2}$$

- In contrast: ***scale symmetry breaking introduces a new scale*** L**.**

- What is the ***physical significance of the new scale?***

The Scales

- The ***nearly*** extreme black hole entropy:

$$S = S_0 + CT$$

- For extremal black holes with $\mathrm{AdS}_2 \times S^2$ near horizon geometry: the S^2 has scale ℓ_2 as well so the ground state entropy

$$S_0 = \frac{4\pi\ell_2^2}{4G_4} = \frac{2\pi}{\kappa_2^2} \; ,$$

  There is ***no scale, just a large dimensionless number***.

- The symmetry breaking scale is the ***specific heat*** $C = 2L$**.**

- Literature: the ***symmetry breaking scale is universal***:

$$C = \frac{\ell_2}{\kappa_2^2}$$

  .

This Talk

- The ***symmetry breaking scale is not universal***
- There are multiple near horizon scales.
- They depend on the charges of the black hole.
- They also depend on boundary values on scalar fields far from the black hole.
- However, there is ***an attractor mechanism*** so these intricate scales can be computed without finding the black hole geometry.

The Extremal Attractor Mechanism

- Setting: a BPS black hole in $\mathcal{N} \geq 2$ supergravity.
- Black hole parameters: charges (p^I, q_I) and scalars at infinity z^i_∞.
- Scalar ***flow***: scalars depend on position $z^i(r)$, approaching $z^i_{\rm hor}$ at the horizon.
- Attractor behavior: the attractor value $z^i_{\rm hor} = z^i_{\rm hor}(p^I, q_I)$ is ***independent of "initial" conditions z^i_∞.***
- Application: ***internal structure of the black hole is independent of coupling constants.***

Near Extreme Black Holes

- Black holes only ***nearly*** extremal so scalars depart from their attractor value.

- $nAdS_2/nCFT_1$ considers the entire ***near horizon region*** and ***scalars are not constant***.

- These ***features introduce new scale***(s).

General Heat Capacity

- Setting: $\mathcal{N} \geq 2$ supergravity in 4D with arbitrary prepotential.

- *Ansatz* with radial symmetry

$$ds_4^2 = g_{\mu\nu} dx^\mu dx^\nu + R^2(r) d\Omega_2^2 \; .$$

- A general ***formula for heat capacity:***

$$L = \frac{1}{2} C = \frac{2\pi^2}{G_4} R^2 \left. \frac{\partial R^2}{\partial r} \right|_{\rm hor} \; .$$

- So: the breaking scale is not related to $\ell_2 = R$ (like the entropy) but ***the derivative of entropy***.

- Generally these two scales are unrelated.

General Flow Equations

- Strategy: analyze all equations of motion.
- Recover standard results for extremal black holes.
- Develop ***perturbation theory*** to relax extremality condition.
- Details: somewhat messy.
- Results: easily summarized by simple ***extremization principles***.

The Extremal Attractor

- The ***spacetime central charge*** $\mathcal{Z}$ is a function of scalars (with charges as parameters):

$$\mathcal{Z}(X^I) = e^{\mathcal{K}/2}\left(X^I q_I - \frac{\partial F}{\partial X^I} p^I\right) \ .$$

  X^I are (projective) scalars, $F = F(X^I)$ the prepotential, $\mathcal{K}$ the Kähler potential.

- The $\mathcal{Z}$ acts like ***an effective potential***: physical values of scalars at the horizon z^i_{hor} are ***determined by its extrema***.

- Note: computes z^i_{hor} for general charges ***without constructing the black hole solution***.

The Entropy

- The extremal entropy is also given the extremization principle:

$$S_{\rm ext} = \pi |\mathcal{Z}|^2_{z^i = z^i_{\rm hor}} \ .$$

- The ***near extremal entropy***:

$$S = S_{\rm ext} + CT \ .$$

- Intuition: expect $C \sim S\partial_r S$ with a ***"radially dependent"*** entropy S.

- Also expect $S \sim |\mathcal{Z}|^2$ where $\mathcal{Z}$ is the ***spacetime central charge***.

Near Extremal Attractor

- The entropy function S does not actually depend on position, but it depends on charges.

- We can ***generate a change in position by adjusting charges appropriately***.

- Symplectic invariance (duality) of $\mathcal{N} = 2$ supergravity determines such "motion in charge space" uniquely.

- A duality invariant formula in the language of special geometry:

$$L = \frac{1}{2}C = 8\pi^2 e^{\mathcal{K}/2} \left(X^I_\infty \frac{\partial}{\partial p^I} + \frac{\partial F}{\partial X^I_\infty} \frac{\partial}{\partial q_I} \right) |\mathcal{Z}|^4_{z^i = z^i_{\rm hor}}$$

Explicit Example: The STU Model

- Eg.: $F = \frac{X^1 X^2 X^3}{X^0}$, simplify charges so $p^0 = 0, q_1 = q_2 = q_3 = 0$.

- The ***central charge is the sum of constituent masses***

$$\mathcal{Z}(X^I) = \frac{q_0}{R} + T_5 R \left(p^1 \mathrm{Vol}[\mathcal{P}_1] + p^2 \mathrm{Vol}[\mathcal{P}_2] + p^3 \mathrm{Vol}[\mathcal{P}_3] \right) \ .$$

  $p^{1,2,3}$ are M5-brane numbers, $\mathcal{P}_{1,2,3}$ are 4-cycles
  q_0 is momentum quantum number, R is ***radius of*** S^1 ***at infinity***.

- The ***extremal*** attractor mechanism: R ***at the horizon*** is

$$R_{\mathrm{hor}} = \sqrt{\frac{q_0}{p^1 p^2 p^3}}\, l_s$$

  independently of its asymptotic value.

The Entropy

- The extremal entropy

$$S = \pi |\mathcal{Z}|^2_{z^i = z^i_{\rm hor}} = 2\pi \sqrt{q_0 p^1 p^2 p^3}$$

- The ***symmetry breaking scale***/near-extreme entropy:

$$L = 8\pi^2 e^{\mathcal{K}/2} \left(X^I_\infty \frac{\partial}{\partial p^I} + \frac{\partial F}{\partial X^I_\infty} \frac{\partial}{\partial q_I} \right) |\mathcal{Z}|^4_{z^i = z^i_{\rm hor}}$$

$$= 2\pi q_0 p^1 p^2 p^3 \left(\frac{R}{q_0} + \frac{1}{T_5 R} \sum_{i=1,2,3} \frac{1}{p^i {\rm Vol}[\mathcal{P}_i]} \right)$$

- It ***depends on moduli at infinity***.

- It depends on ***non-trivial combinations of charges***.

The Long String Scale

- In the ***dilute gas regime*** the excitation energy (momenta) are ***small compared to background*** (M5-branes).

- Then the symmetry breaking scale is

$$L = 2\pi p^1 p^2 p^3 R$$

- This is the ***long string scale*** known from microscopic black hole models.

- Physics: low energy excitations "live" on a circle of length L rather than on a circle of radius R.

$\mathrm{nAdS_2/nCFT_1}$ from $\mathrm{AdS_3/CFT_2}$?

- The dilute gas regime is equivalent to the ***Cardy regime***.
- $\mathrm{CFT_2}$ language: large central charge $c = 6p^1p^2p^3 \gg 1$ but energy is "fractionated" in units of $2\pi/L$ so numerous excitations anyway.
- Entropy in Cardy regime:

$$S = 2\pi\sqrt{\frac{1}{6}ch}$$

- In this limit: $\mathrm{nAdS_2/nCFT_1}$ is inherited from $\mathrm{AdS_3/CFT_2}$
- But ***this is a very special case***: $\mathrm{nAdS_2/nCFT_1}$ applies for any relative size of the four charges.
- Example: near extreme Reissner-Nordström black holes are "equal charge" rather than "dilute gas" (large hiererchy).

Who is the Dilaton?

- The default geometry

$$ds_4^2 = g_{\mu\nu} dx^\mu dx^\nu + R^2 d\Omega_2^2 \, .$$

- The S^2 radius R is “the” dilaton in simple cases (Jackiw-Teitelboim).

- But 2D theory from $\mathcal{N} = 2$ SUGRA has many other scalar fields.

- In general ***all scalar fields are important.***

A Flow of Many Fields

- Near the horizon $R^2 \sim |\mathcal{Z}|^2$.

- "The" breaking scale is the (roughly) the radial derivative of $|\mathcal{Z}|^2$

- Other scalar fields ***approach*** their fixed value $z^i_{\rm hor}$ at the horizon.

- The "radial derivative" In the near horizon region is equivalent to amounts to "motion in charge space"

$$z^i = z^i_{\rm hor}\left(1 + r D^i_\infty |\mathcal{Z}|^2_{z^i = z^i_{\rm hor}}\right)$$

- There are ***many scales*** but they are ***all determined by an extremization principle***.

Summary

- nAdS$_2$/nCFT$_1$ is a new precise holography that depends on an intrinsic scale.

- It studies the ***approach*** to extremality. My point: it ***depends on the "direction" of approach***.

- The details can be elaborate but they are ***determined by an attractor mechanism*** (in $\mathcal{N} = 2$ SUGRA) .

- Limits of this work: radial symmetry, $D = 4$, GR, near BPS assumed in last part.

- Future: rotating black holes, $D \neq 4$, higher derivatives, nonBPS branch.