BMS Supertranslation Symmetry Implies Faddeev-Kulish Amplitude

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Based on...

"BMS supertranslation symmetry implies Faddeev-Kulish amplitude"

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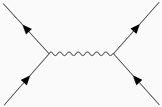
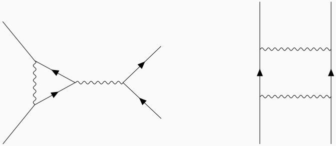
Background

Consider a 2-to-2 scattering amplitude $\langle q_1, q_2|\mathcal{S}|p_1, p_2\rangle$ in QED.

At lowest order, all is well:



With loops, diagrams have infrared divergences.



These divergences exponentiate, and the amplitude vanishes in the limit where the infrared regulator is removed:

$$\langle q_1, q_2|\mathcal{S}|p_1, p_2\rangle = 0$$

Background

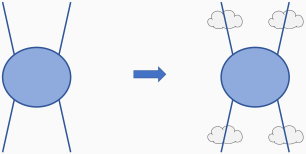
Traditionally, this problem has been circumvented at the level of *cross section* via the Bloch-Nordsieck method; the *S-matrix elements* are left ill-defined.

An alternative: replace Fock states with the dressed (Faddeev-Kulish, FK) states:

$$|\mathbf{p}\rangle \quad \rightarrow \quad e^{R(\mathbf{p})}\,|\mathbf{p}\rangle\,,$$

where $R(\mathbf{p})$ is an anti-Hermitian operator which, for gravity, is given as

$$R(\mathbf{p}) = \int_{\text{soft}} \frac{d^3k}{(2\pi)^3(2\omega_{\mathbf{k}})} f^{\mu\nu}(\mathbf{p},\mathbf{k}) \left(a^{\dagger}_{\mu\nu}(\mathbf{k}) - a_{\mu\nu}(\mathbf{k})\right).$$



$\langle\text{out}|\mathcal{S}|\text{in}\rangle$ $\qquad$ $\langle\text{out}|e^{-R}\mathcal{S}e^{R}|\text{in}\rangle$

Amplitudes built using FK states (FK amplitudes) are free of infrared divergences.

Motivation

Gauge/gravity theories have asymptotic symmetries:

- Large gauge symmetry for QED.
- BMS symmetry for gravity.

Charges of asymptotic symmetries should be conserved:

$$\langle \text{out}| \, [Q, \mathcal{S}] \, |\text{in}\rangle = 0.$$

However, Fock states are not charge eigenstates. Scattering amplitudes built with Fock states violate charge conservation and therefore vanish – this is reflected in infrared divergences. [Kapec, Perry, Raclariu, Strominger '17]

$\star$ Recall that FK amplitudes are free of infrared divergence – this hints at a close relation between the FK states and the asymptotic symmetries. ([Gabai, Sever '16] for QED, [Choi, Kol, Akhoury '17] for gravity.)

BMS Supertranslation Charge

There is a BMS supertranslation charge $Q(f)$ for each 2-sphere function $f = f(w, \bar{w})$.

$$Q(f) = Q_S(f) + Q_H(f)$$

The action of the hard charge Q_H on a Fock state is

$$Q_H \left|\mathbf{p}\right\rangle = -\int \frac{d^2w}{2\pi} \frac{(\epsilon^+(w,\bar{w}) \cdot p)^2}{p \cdot \hat{x}_w} D_{\bar{w}}^2 f(w,\bar{w}) \left|\mathbf{p}\right\rangle .$$

The soft charge Q_S is given as

$$Q_S = -\frac{1}{8\pi G} \int du\, d^2w\, \gamma_{w\bar{w}} N^{\bar{w}\bar{w}} D_{\bar{w}}^2 f(w,\bar{w}).$$

The Bondi news tensor $N^{\bar{w}\bar{w}}$ contains zero-mode graviton operators.

BMS Supertranslation Charge

The FK states are charge eigenstates of the BMS supertranslation:

$$Qe^{R(\mathbf{p})}\left|\mathbf{p}\right\rangle = C(\mathbf{p})e^{R(\mathbf{p})}\left|\mathbf{p}\right\rangle .$$

Charge conservation demands $\sum_{i\in\text{out}} C(\mathbf{p}_i) - \sum_{i\in\text{in}} C(\mathbf{p}_i) = 0$. Here $C(\mathbf{p}) \propto p$; conservation automatically follows from energy-momentum conservation.

In fact, any coherent state of the form,

$$\exp\left\{\int \frac{d^3k}{(2\pi)^3(2\omega_{\mathbf{k}})} N^{\mu\nu}(a^\dagger_{\mu\nu} - a_{\mu\nu})\right\}\left|\mathbf{p}\right\rangle ,$$

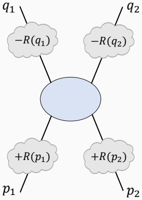
where $N^{\mu\nu} = O(1/\omega_{\mathbf{k}})$ is a charge eigenstate, and charge conservation demands

$$N^{\mu\nu}_{\text{out}} - N^{\mu\nu}_{\text{in}} = \sqrt{8\pi G}\left[\sum_{i\in\text{out}} \frac{p_i^\mu p_i^\nu}{p_i\cdot k} - \sum_{i\in\text{in}} \frac{p_i^\mu p_i^\nu}{p_i\cdot k}\right]$$

$\implies$ There exists a broader class of dressed states (containing the set of FK states) that conserve the supertranslation charge.

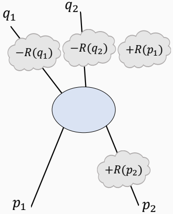
BMS Supertranslation Charge

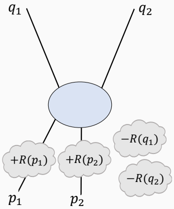
A 2-to-2 FK amplitude looks like



$$= \langle q_1, q_2 |\, e^{-R(q_1)-R(q_2)} \mathcal{S} e^{R(p_1)+R(p_2)} \,| p_1, p_2 \rangle \,.$$

Examples of other amplitudes that conserve supertranslation charge are:





$\langle f | e^{-R(q_1)-R(q_2)+R(p_1)} \mathcal{S} e^{R(p_2)} | i \rangle$ $\quad$ $\langle f | \mathcal{S} e^{R(p_1)+R(p_2)-R(q_1)-R(q_2)} | i \rangle$

But FK amplitudes are infrared-finite! Are the latter amplitudes also infrared-finite?

$\Longrightarrow$ Conjectured to be true in [Kapec, Perry, Raclariu, Strominger '17].

Infrared-finiteness

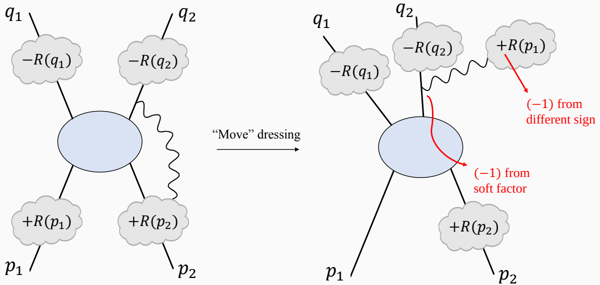
It turns out that they are!

We have an explicit formula for the leading term of a scattering amplitude with N (N') absorbed (emitted) virtual gravitons [Choi, Kol, Akhoury '17]:

$$(-1)^N \left[\prod_{r=1}^{N+N'} \int \frac{d^3 k_r}{(2\pi)^3 (2\omega_r)} f_{\mu\nu} I^{\mu\nu,\rho_r\sigma_r} \right] \mathcal{J}_{\rho_1\sigma_1\cdots\rho_{N+N'}\sigma_{N+N'}}$$

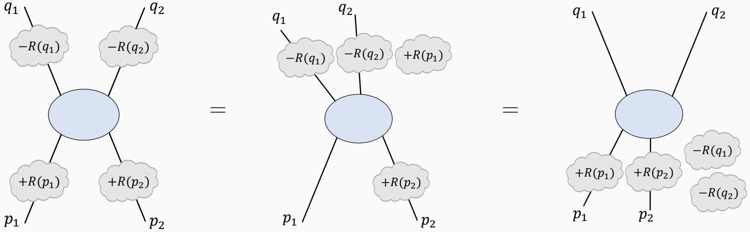
where $I^{\mu\nu,\rho\sigma} = \frac{1}{2}(\eta^{\mu\rho}\eta^{\nu\sigma} + \eta^{\mu\sigma}\eta^{\nu\rho} - \eta^{\mu\nu}\eta^{\rho\sigma})$, and $\mathcal{J}_{\dots}$ is some complicated tensor.

The net effect of "moving" a dressing from the in-state to the out-state can be summarized in the following diagram:



"Moving" the dressing has no net effect on the leading term of the amplitude.

Therefore,



Since FK amplitude is infrared-finite, all amplitudes that conserve BMS supertranslation charge are infrared-finite. This proves the conjecture of [Kapec, Perry, Raclariu, Strominger '17].

Summary

To summarize the main points:

- Conventional S-matrix elements vanish due to infrared divergences. This is a penalty for violating charge conservation of the asymptotic symmetries.
- FK amplitudes are well defined – i.e. they do not exhibit infrared divergence.
- There thus is a close connection between asymptotic symmetries and FK states: The set of FK states is a subset of charge eigenstates that automatically conserve the charge of asymptotic symmetry.
- However, any amplitude that conserves the charge (and therefore is non-zero) is equivalent to the corresponding FK amplitude at the leading order.