Gravitational Decoupling II:
Picard-Lefschetz Theory

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Overview

A Lorentzian prescription for semiclassical physics:

- Review of Picard-Lefschetz theory
- Tunneling with Picard-Lefschetz theory between non-metastable states
- 'Euclidean' solutions with Picard-Lefschetz theory
- Gravitational decoupling in the Picard-Lefschetz prescription vs. the Euclidean prescription
- Conclusions

A Lorentzian Prescription: Picard-Lefschetz

- The Euclidean prescription generically leads to a failure of decoupling
- Instead we should use a Lorentzian prescription: Picard-Lefschetz

  Witten '11

- Euclideanization: complexify time

$$Z = \int \mathcal{D}\phi \, e^{iS} \to \int \mathcal{D}\phi \, e^{-S_E}$$

- Picard-Lefschetz: complexify fields

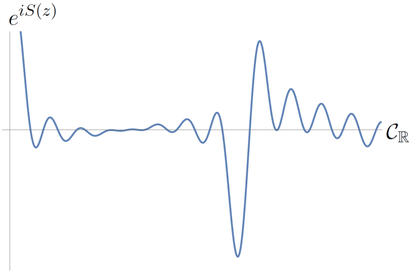
$$Z = \int_{\mathcal{C}_{\mathbb{R}}} \mathcal{D}\phi \, e^{iS} \to \int_{\mathcal{J}_\sigma} \mathcal{D}\phi \, e^{iS}$$

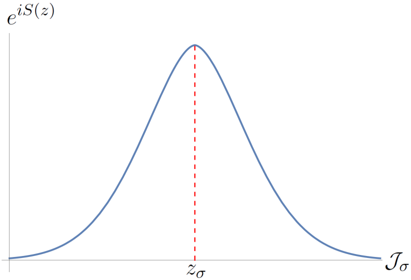
A Lorentzian Prescription: Picard-Lefschetz

- Start with 1d integral

$$Z(\lambda) = \int_{\mathcal{C}_{\mathbb{R}}} dz\, e^{iS(z)/\lambda}$$

- Define the exponent $\mathcal{I} \equiv iS(z)/\lambda = h + iH$
- Find a contour with $\lim_{z\to\infty} h \to -\infty$ and $H = \text{const}$
- These are the steepest descent contours $\mathcal{J}_\sigma$ around saddle points z_σ





A Lorentzian Prescription: Picard-Lefschetz

- Enumerate all saddle points and define the Lefschetz decomposition

$$\int_{\mathcal{C}_{\mathbb{R}}} dz\; e^{iS(z)/\lambda} = \sum_{\sigma} n_\sigma \int_{\mathcal{J}_\sigma} dz\; e^{iS(z)/\lambda}$$

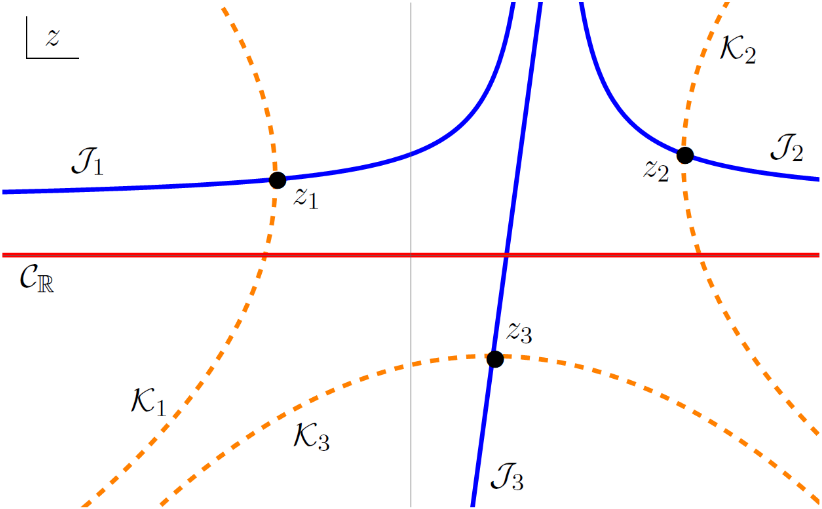
- The n_σ are topologically determined intersection numbers depending on steepest ascent contours $\mathcal{K}_\sigma$

$$n_\sigma = \langle \mathcal{C}_{\mathbb{R}}, \mathcal{K}_\sigma \rangle \mod 2$$

- Claim: Lefschetz decomposition defines a semiclassical expansion for the path integral

$$Z(\lambda) = \int_{\mathcal{C}_{\mathbb{R}}} \mathcal{D}\phi\; e^{iS[\phi]/\lambda} = \sum_{\sigma} n_\sigma \int_{\mathcal{J}_\sigma} \mathcal{D}\phi\; e^{iS[\phi]/\lambda}$$
$$= \sum_{\sigma} n_\sigma e^{iS[\phi_\sigma]/\lambda} \sum_{j} a_{\sigma,j} \lambda^j$$

A Lorentzian Prescription: Picard-Lefschetz



$$n_1 = n_2 = 1 \qquad n_3 = 0$$

Tunneling with Picard-Lefschetz

- To illustrate PL consider the model Garay, Halliwell, Mena Marugan '91

$$S_g = \int d^4x \sqrt{-g} \left(\frac{M_p^2}{2} \mathcal{R} - \frac{1}{2} \left(\partial \phi \right)^2 - \sqrt{\frac{3}{2}} \alpha \cosh \left(\sqrt{\frac{2}{3}} \frac{\phi}{M_p} \right) \right) + S_{GHY}$$

- Take a minisuperspace ansatz

$$ds^2 = -\frac{N^2}{a\left(t\right)^2} dt^2 + a\left(t\right)^2 d\Omega_3^2$$

- Define $t \in [0, 1]$ and physical time $d\tau = \frac{N}{a} dt$
- Choosing the gauge $\dot{N} = 0$, the path integral takes the form

$$Z = \int_{\mathcal{C}_{\mathbb{R}}} dN \int \mathcal{D}a \mathcal{D}\phi \, e^{iS_g}$$

Tunneling with Picard-Lefschetz

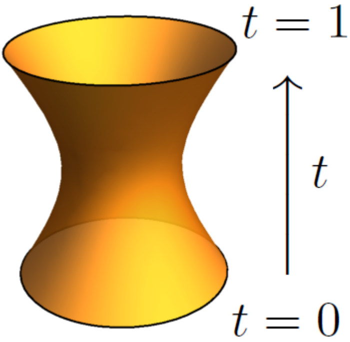
- The decoupled theory is free

$$V_d\left(\phi\right) = \lim_{M_p \to \infty} \sqrt{\frac{3}{2}} \alpha \cosh\left(\sqrt{\frac{2}{3}} \frac{\phi}{M_p}\right) = \sqrt{\frac{3}{2}} \alpha$$

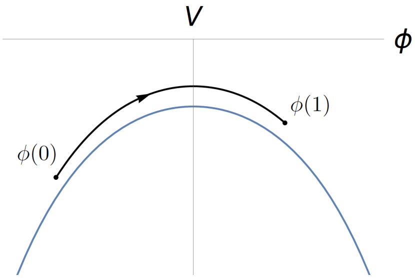
- Given arbitrary boundary conditions

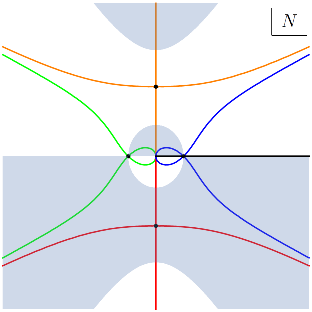
$$a\left(0\right) = a_1 \qquad \phi\left(0\right) = \phi_1$$
$$a\left(1\right) = a_2 \qquad \phi\left(1\right) = \phi_2$$

- There are four saddle points labeled $N_{\pm\pm}$
- First example: let $a_1 = a_2$



Tunneling with Picard-Lefschetz



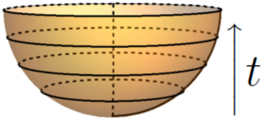


- Only the saddle point N_{++} (in blue) contributes with action

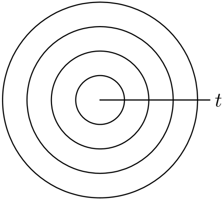
$$\frac{S_g\left[N_{++}\right]}{\mathcal{V}} = \frac{\left(\phi_2 - \phi_1\right)^2}{2\Delta\tau} + \mathcal{O}\left(M_p^{-1}\right)$$

- In the decoupling limit this is the action for a free theory: decoupling is successful
- For $\alpha < 0$ this can be viewed as the field tunneling through the tip of an inverted cosh

'Euclidean' Solutions with Picard-Lefschetz



$\xrightarrow{M_p \to \infty}$



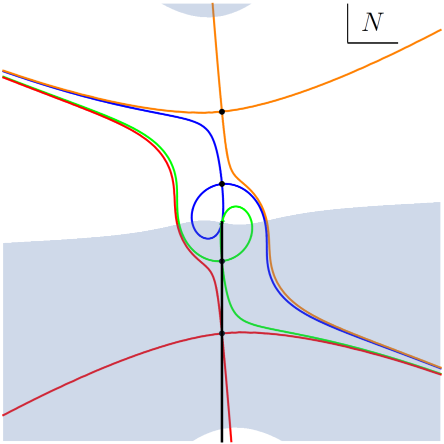
- Second example: reproduce Euclidean solutions with PL
- Assume boundary conditions $a_1 = 0$ and $a_2 > 0$
- Decoupled theory is Euclidean with metric $ds^2 = dt^2 + t^2 d\Omega_3^2$
- The potential is constant and so there is only a constant solution $\phi(t) = \text{const}$
- The action per unit spacetime volume is then

$$\frac{S_{d,E}}{\mathcal{V}} = \sqrt{\frac{3}{2}}\alpha$$

‘Euclidean’ Solutions with Picard-Lefschetz

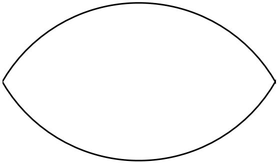
$$\frac{S_g[N_{-+}]}{\mathcal{V}} = \frac{12iM_p^2}{a_2^2} + \mathcal{O}\left(M_p^0\right)$$

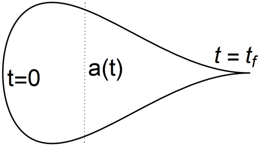
$$\frac{S_g[N_{--}]}{\mathcal{V}} = i\sqrt{\frac{3}{2}}\alpha + \mathcal{O}\left(M_p^{-1}\right)$$



- With gravity two saddle points contribute: N_{-+} (green) and N_{--} (red)
- In the decoupling limit N_{-+} ceases to contribute and N_{--} agrees exactly with the decoupled theory

‘Euclidean’ Solutions with Picard-Lefschetz





- Solutions found with PL prescription have conical singularities at the poles
- This means $\frac{\partial \phi}{\partial \tau} \neq 0$ and so these are excluded by the Euclidean prescription
- Conical singularity does not spoil decoupling
- Smoothing out one pole necessarily creates an HT singularity at the opposite pole

Euclidean prescription requires solutions that are pathological and forbids solutions that are not!

Conclusions

- The Euclidean prescription is not an indiscriminate tool for tunneling computations
- Generically, the Euclidean prescription leads to a failure of decoupling due to HT-type singularities
- The Euclidean prescription severely restricts the allowed boundary conditions
- A potential alternative Lorentzian prescription is Picard-Lefschetz
- PL typically allows for successful decoupling
- The solutions in PL for which decoupling succeeds are generically the solutions forbidden by the Euclidean prescription