Precision Holography with Supersymmetric Wilson Loops

Vimal Rathee
Graduate Student, University of Michigan

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1802.03016, 1802.06789 J. Aguilera Damia, A. Faraggi, L. Pando Zayas, V. Rathee, G. A Silva

Motivation

- AdS/CFT beyond the leading order:

$$Z_{Field\ Theory}(\phi_0) = Z_{String} \approx \exp\left(-S(\phi \to \phi_0)_{Grav}\right).$$

- Localization: A plethora of exact results for supersymmetric field theories.

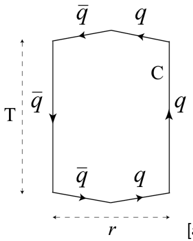
- What can we learn about string perturbation theory given the "experimental data" provided by localization?

Introduction

- Wilson Loops (WL) are an important class of gauge invariant non-local operators.

$$W_R\,(\mathrm{C}) = tr_R\,\mathcal{P}\,\mathrm{exp}\,\oint_C (A_\mu\, dx^\mu)$$

- The expectation value measures the effective action of an external particle; order parameter for confinement, $V_{q\bar{q}} \propto r$.



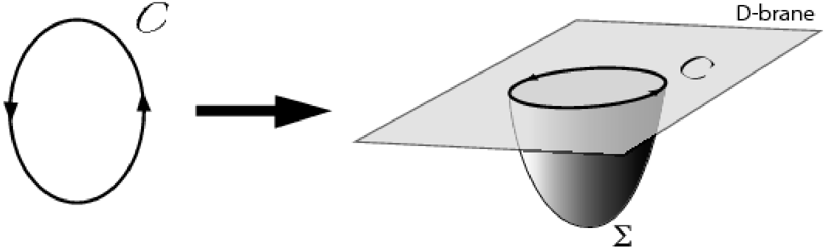
[arxiv:1412.8008]

- Wilson loop in Gauge/Gravity Correspondence
- The contour becomes a surface in higher dimensions.
- Expectation value:

$$\langle W(\mathcal{C})\rangle = Z_{\text{string}}\left(\partial\Sigma = \mathcal{C}\right)$$

- Right regime

$$Z_{\text{string}}\left(\partial\Sigma = \mathcal{C}\right) = e^{-S(\mathcal{C})}$$



[arxiv:1310.4319]

Matrix Model: Half BPS Wilson Loop in $\mathcal{N} = 4$ SYM

- The Matrix Model computation gives the exact answer, for any N and λ, in terms of Laguerre polynomial,

$$\begin{aligned}
\langle W_\Box \rangle_{\text{circle}} &= \frac{1}{N} L^1_{N-1}\left(-\frac{\lambda}{4N}\right) e^{\lambda/8N} \\
&\approx \frac{2}{\sqrt{\lambda}} I_1(\sqrt{\lambda}) + \frac{\lambda}{38N^2} I_2(\sqrt{\lambda}) + \frac{\lambda^2}{1280N^4} I_4(\sqrt{\lambda}) + \ldots \\
&\approx \exp\left(\sqrt{\lambda} - \frac{3}{4}\ln\lambda - \frac{1}{2}\ln\frac{\pi}{2} + \ldots\right)
\end{aligned}$$

Gravity Side: Beyond the leading order

- Forste-Ghoshal-Theisen 9903042, Drukker-Gross-Tseytlin 0001204, Kruczenski-Tirziu 0803.0315, Buchbinder-Tseytlin 1404.4952.

$$\begin{aligned} \langle W \rangle &= \exp(-\Gamma), \qquad \Gamma = \Gamma_0 + \Gamma_1, \\ \Gamma_1 &= \frac{1}{2} \ln \frac{[\det(-\nabla^2 + 2)]^3 [\det(-\nabla^2)]^5}{[\det(-\nabla^2 + \frac{1}{4} R^{(2)} + 1)]^8} \end{aligned}$$

- Five massless modes (S^5); three massive modes $AdS_2 \subset AdS_5$.

$$\langle W_{\Box} \rangle_{\text{circle}} = \exp\left(\sqrt{\lambda} - \frac{1}{2} \ln(2\pi) + \ldots \right)$$

WL beyond the leading order: Problem/Opportunity

$$\langle W_{\Box}\rangle_{\text{circle}} = \exp\left(\sqrt{\lambda} - \frac{3}{4}\ln\lambda - \frac{1}{2}\ln\frac{\pi}{2} + \dots\right)$$

$$\langle W_{\Box}\rangle_{\text{circle}} = \exp\left(\sqrt{\lambda} - \frac{1}{2}\ln(2\pi) + \dots\right)$$

- Missing the $\ln(\lambda)$ term on the gravity side (zero modes, etc.).
- Numerical discrepancy is not an error: Drukker-Gross-Tseytlin 0001204, Kruczenski-Tirziu 0803.0315, Buchbinder-Tseytlin 1404.4952.

Toward a Precision Computation

- The AdS/CFT formula:

$$< W(C) >_{CFT} = < W(\Sigma \to C) >_{String}$$

- What we are doing:

$$Z_{string} = exp(-S_{classical}) \times Z_{1-loop} \times (\text{Topological Sector})$$

- We are missing aspects of string perturbation theory: Ghost zero modes, etc.
- $\Longrightarrow$ Compare configurations with the same world sheet topology!
- The $1/4$ BPS Wilson loop beyond the leading order.

Holographic 1/4 BPS WL

- Classical solution – Ansatz:

$$\psi = \tau, \qquad \sinh\rho = \frac{1}{\sinh\sigma},$$
$$u = 0$$
$$\phi = \tau, \qquad \sin\theta = \frac{1}{\cosh(\sigma_0 + \sigma)},$$

- Classical solution – World-sheet metric:

$$ds^2 = \left(\frac{1}{\sinh^2\sigma} + \frac{1}{\cosh^2(\sigma_0 - \sigma)}\right)(d\tau^2 + d\sigma^2).$$

Fluctuations

Forini-Giangreco-Griguolo-Seminaro-Vescovi [1512.00841],
Faraggi-Pando Zayas-Silva-Trancanelli [1601.04708]

- The quadratic action for the bosonic fluctuations is

$$S^{2,3,4} = \frac{\sqrt{\lambda}}{4\pi} \int d\tau d\sigma \sqrt{g} \left(g^{ab} \partial_a \chi \partial_b \chi + \frac{2}{\sqrt{g}} \chi^2 \right) ,$$

$$S^{5,6} = \frac{\sqrt{\lambda}}{2\pi} \int d\tau d\sigma \sqrt{g} \left(g^{ab} D_a \chi (D_b \chi)^\dagger - \frac{2m^2(\sigma_0)}{\sqrt{g}} |\chi|^2 \right) ,$$

$$S^{7,8,9} = \frac{\sqrt{\lambda}}{4\pi} \int d\tau d\sigma \sqrt{g} \left(g^{ab} \partial_a \chi \partial_b \chi - \frac{2\sin^2\theta}{\sqrt{g}} \chi^2 \right) .$$

One-loop effective action

$$e^{-\Gamma^{1-\text{loop}}_{\text{effective}}} = \frac{\left(\text{Det}\,\mathcal{O}^{+}\right)^{\frac{4}{2}}\ \left(\text{Det}\,\mathcal{O}^{-}\right)^{\frac{4}{2}}}{\left(\text{Det}\,\mathcal{O}^{2,3,4}\right)^{\frac{3}{2}}\ \left(\text{Det}\,\mathcal{O}^{5,6}\right)^{\frac{2}{2}}\ \left(\text{Det}\,\mathcal{O}^{7,8,9}\right)^{\frac{3}{2}}}\,,$$

$$\ln\left(\text{Det}\,\mathcal{O}\right) = \sum_{E} \ln\left(\text{Det}\,\mathcal{O}_E\right)\ ,$$

- $\mathcal{O}_E$ is the corresponding one-dimensional operator acting on a specific Fourier mode.

Ratio of 1-loop effective action

- For $1/4$ BPS dependence on the value of σ_0 that characterizes the classical string solution. The 1/2 BPS is $\sigma_0 \to \infty$.

$$\Omega_E^{2,3,4}(\sigma_0) = \ln\left[\frac{\text{Det }\mathcal{O}_E^{2,3,4}(\sigma_0)}{\text{Det }\mathcal{O}_E^{2,3,4}(\infty)}\right] ,$$

$$\Omega_E^{5,6}(\sigma_0) = \ln\left[\frac{\text{Det }\mathcal{O}_E^{5,6}(\sigma_0)}{\text{Det }\mathcal{O}_E^{5,6}(\infty)}\right] ,$$

$$\Omega_E^{7,8,9}(\sigma_0) = \ln\left[\frac{\text{Det }\mathcal{O}_E^{7,8,9}(\sigma_0)}{\text{Det }\mathcal{O}_E^{7,8,9}(\infty)}\right] ,$$

$$\Omega_E^{\alpha}(\sigma_0) = \ln\left[\frac{\text{Det }\mathcal{O}_E^{\alpha}(\sigma_0)}{\text{Det }\mathcal{O}_E^{\alpha}(\infty)}\right]$$

- Each ratio is to be computed using the Gelfand-Yaglom (Coleman) method.

- Forini-Giangreco-Griguolo-Seminaro-Vescovi [1512.00841], Faraggi-Pando Zayas-Silva-Trancanelli [1601.04708]

$$\Delta\Gamma^{1-\text{loop}}_{\text{effective}}(\sigma_0) = \frac{1}{2}\sum_{E\in\mathbb{Z}}\Big(3\Omega_E^{2,3,4}(\sigma_0) + 2\Omega_E^{5,6}(\sigma_0) + 3\Omega_E^{7,8,9}(\sigma_0)\Big) - \frac{4}{2}\sum_{E\in\mathbb{Z}+\frac{1}{2}}\left(\Omega_E^{+}(\sigma_0) + \Omega_E^{-}(\sigma_0)\right) .$$

- Result

$$\Delta\Gamma^{1-\text{loop}}_{\text{effective}} = \frac{3}{2}\ln\tanh\sigma_0 - \ln\sqrt{\frac{1+\tanh\sigma_0}{2}} = \frac{3}{2}\ln\cos\theta_0 - \ln\cos\frac{\theta_0}{2} ,$$

The first term gives the predicted result from the gauge theory. It comes from the $E = 0$ mode of the $\Omega_E^{7,8,9}$ determinant.

Revision 1

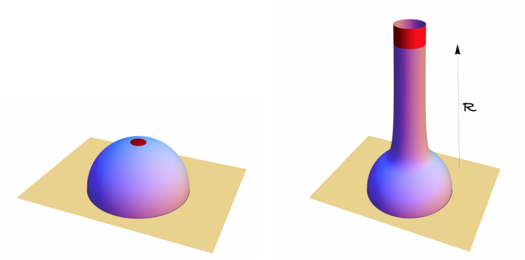
- Forini-Tseytlin-Vescovi [1702.02164] have proposed a perturbative heat kernel computation that matches the field theory answer.
- Hard to compute heat kernels in non-homogeneous spaces. Expand the heat kernel around $\theta_0 = 0$.

$$
\begin{aligned}
g_{ij} &= g_{ij}^{AdS_2} + \theta_0^2\, \tilde{g}_{ij} + O(\theta_0^4), \\
\mathcal{O} &= \mathcal{O}^{AdS_2} + \theta_0^2\, \tilde{\mathcal{O}} + O(\theta_0^4), \\
K_{\mathcal{O}}(x, x'; t) &= K_{\mathcal{O}}^{AdS_2}(x, x'; t) + \theta_0^2\, \tilde{K}_{\mathcal{O}}(x, x'; t) + O(\theta_0^4),
\end{aligned}
$$

- What does it teach us? The Gelfand-Yaglom method might be imposing very strict conditions.

Revision 2

- Cagnazzo-Medina Rincon-Zarembo [1712.07730]
- Mapping problem to cylinder



[arxiv:1712.07730]

- Diffeomorphism-invariant Regulator

$$R \equiv R_{\text{inv}} - \ln \cos \frac{\theta_0}{2}$$

- Well defined problem on disk.

Zeta-function Regularization

Motivation from Dunne-Kirsten [0607066], Non-perturbative result for one-loop corrections.

$$\ln \frac{\text{det}\mathcal{O}}{\text{det}\mathcal{O}^{\text{free}}} = -\ln\left(\mu^2\right)\hat{\zeta}_{\mathcal{O}}(0) - \hat{\zeta}'_{\mathcal{O}}(0), \qquad \hat{\zeta}_{\mathcal{O}}(s) \equiv \zeta_{\mathcal{O}}(s) - \zeta_{\text{free}}(s),$$

where μ is renormalization parameter.

- Induced Geometry $ds^2_M = M\, ds^2_{AdS_2} = M(\rho)\left(d\rho^2 + \sinh^2\rho\, d\tau^2\right)$
- Fluctuation operators

$$\begin{aligned} \mathcal{O}_M &= M^{-1}\,\mathcal{O}_{AdS_2}, \quad \text{(bosons)} \\ \mathcal{O}_M &= M^{-1/2}\,\mathcal{O}_{AdS_2}, \quad \text{(fermions)} \end{aligned}$$

$$\mathcal{O}^{B}_{AdS_2} = -g^{\mu\nu}\, D_\mu D_\nu + m^2 + V, \qquad D_\mu = \nabla_\mu - \iota q\, \mathcal{A}_\mu .$$

$$\ln \frac{\det \mathcal{O}_{AdS_2}}{\det \mathcal{O}^{\text{free}}_{AdS_2}} = \ln \frac{\det \mathcal{O}_0}{\det \mathcal{O}^{\text{free}}_0} + \sum_{l=1}^{\infty} \left(\ln \frac{\det \mathcal{O}_l}{\det \mathcal{O}^{\text{free}}_l} + \ln \frac{\det \mathcal{O}_{-l}}{\det \mathcal{O}^{\text{free}}_{-l}} + \frac{2}{l}\, \hat{\zeta}_{\mathcal{O}}(0) \right)$$
$$- 2\left(\gamma + \ln \frac{\mu}{2}\right) \hat{\zeta}_{\mathcal{O}}(0) + \int_0^\infty d\rho\, \sinh\rho\, \ln(\sinh\rho)\, V$$
$$- q^2 \int_0^\infty d\rho \frac{\mathcal{A}^2}{\sinh\rho},$$

$$\hat{\zeta}_{\mathcal{O}}(0) = -\frac{1}{2} \int_0^\infty d\rho\, \sinh\rho\, V,$$

Anomaly :

$$\ln \frac{\det \mathcal{O}_M}{\det \mathcal{O}^{\text{free}}_M} = \ln \frac{\det \mathcal{O}_{AdS_2}}{\det \mathcal{O}^{\text{free}}_{AdS_2}} + \frac{1}{4\pi} \int d^2\sigma\, \sqrt{g}\, \ln M \left[m^2 + V - \frac{R}{6} + \frac{1}{12} \nabla^2 \ln M \right].$$

Conclusions and Open Directions

- Precision holography is a painstaking job and we are learning how to perform holographic computations.
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- Is there a similar bulk localization mechanism for D-branes (corresponding to Wilson loops in higher dimensional representations)?

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- There are various venues in $\mathcal{N} = 4$ and ABJM.
- Hints of string localization?
- Explicit hints of a localization structure? What are these modes the localization locus of ?
- Is there a similar bulk localization mechanism for D-branes (corresponding to Wilson loops in higher dimensional representations)?
- Similar techniques have been applied by Ashoke Sen and collaboraters to asymptotically flat black holes. We should apply to the entropy of asymptotically AdS black holes.

Thanks !