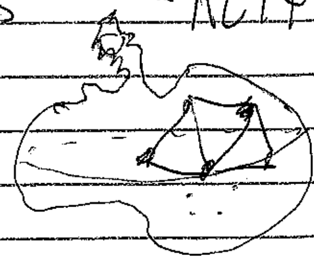
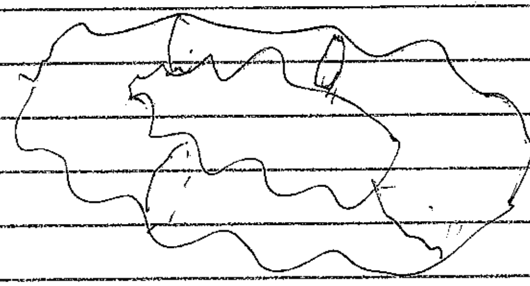
Anomalies - KCTP Journal Club

QFT 1



$\chi = V - E + F$

QFT 2



A crude computation allows you to distinguish 1 from 2 due to topology genus = 0 or 1

Anomalies - a useful tool for understanding some properties of a QFT w/o necessarily being able to compute everything precisely.

General meaning: A symmetry is anomalous if it is a symmetry of the classical theory but is violated by quantum effects.

Why this happens

$$\langle \phi(x_1) \ldots \phi(x_n) \rangle = \int \underbrace{\mathcal{D}\phi}_{\text{measure may not be invariant}} \, \phi(x_1) \ldots \phi(x_n) \, e^{\overbrace{iS[\phi]}^{\text{invariant}}}$$

- Fujikawa pt. of view

There as many kinds of anomalies as there are types of symmetries

chiral anomaly
conformal anomaly
gauge anomaly
diffeomorphism anomaly
parity anomaly
discrete anomaly

~~and there is a long list of applications of anomalies. Let me mention some of most important, weighted towards particle theory.~~

Earliest most basic example

QCD

quark kinetic terms $i\bar{\psi}\partial\psi = i\psi_L^\dagger \sigma \partial \psi_L + i\psi_R^\dagger \bar{\sigma}\partial\psi_R$

mass terms $m\bar{\psi}\psi = m(\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L)$

when $m=0$ have $\psi_L \to e^{i\alpha}\psi_L$ $U(1)_L$

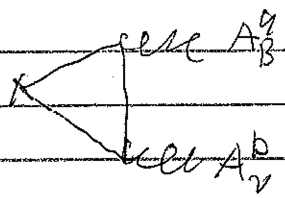
$\psi_R \to e^{i\beta}\psi_R$ $U(1)_R$

or take lin. comb $U(1)_V$ $L+R$ $j^\mu_V = j^\mu_L + j^\mu_R$

$U(1)_A$ $L-R$ $j^\mu_A = j^\mu_L - j^\mu_R$

Classically $\partial_\mu j^\mu{}_A = 0$ when $m=0$

1-loop effects $\partial_\mu j^\mu{}_A = -\frac{N_f}{32\pi^2} \epsilon^{\alpha\beta\mu\nu} F^a_{\alpha\beta} F^a_{\mu\nu}$

From j^μ_A 

The diagram is divergent and can't be regularized in a way that preserves $SO(3)_c$ gauge invariance and conservation of j^μ_A.

Sounds like UV effect.

But ~~a~~ massive fields that can have a mass term consistent w/ a symmetry can have UV div. regularized by Pauli-Villars

∴ anomalies determined by IR behavior - massless fields

Interplay between UV and IR pts of view a common theme in study of anomalies.

Let me mention some applications - first in QCD

- ~~we~~ we know in the real world QCD contains quarks and spontaneously breaks chiral symmetry leading to light N-G bosons - π's and K's

* w/ $N_c = 3$ can prove QCD breaks chiral sym using anomalies - 't Hooft anomaly matching.

* Decay rate $\pi^0 \to \gamma\gamma$ determined by anomalies

* Certain π-K couplings are determined by anomalies - Wess Zumino terms.

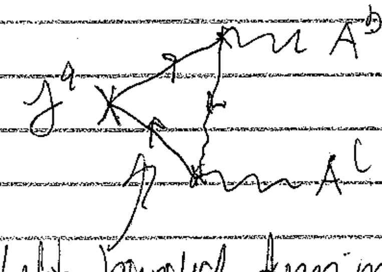
~~More generally in the~~
* In the Skyrmion model of protons as solitons some quantum numbers are fixed by W-Z terms dictated by anomalies.
* anomaly in $U(1)_{baryon}$, $U(1)_{lepton}$ in SM but not ~~Extend~~ in B-L ⇒ turn L into B - leptogenesis.

More generally anomalies are useful in the study of ~~dynamics of~~ strong coupling dynamics and provide constraints on dual descriptions i.e. in Seiberg duals.

* The Standard Model gauges chiral symmetries that distinguish L from R handed fields.

i.e. $\begin{pmatrix}\nu \\ e\end{pmatrix}_L \sim 2$ of $SU(2)_L$ $e_R \sim 1$ of $SU(2)_L$.

In general such theories are inconsistent because gauge inv. is violated at the quantum level. D=4



$\propto \mathrm{Tr}\, \lambda_r^a \{\lambda_r^b, \lambda_r^c\} = d^{abc} = 0$ for real rep.

left-handed fermion in rep r $\neq 0$ for $U(1)$, $SU(n)$ $n \geq 3$, $SO(4n+2)$, E_6 w/ complex reps

SM w/ $G = SU(3) \times SU(2) \times U(1)$
and GUTS w/ $G = SU(5)$, $SO(10)$, E_6
all have potential anomalies

$\Rightarrow$ anomalies restrict model building

Anomalies in other dimensions.

In 2n dimensions have chiral fermions and there is a chiral anomaly

$$\cancel{\delta_A S_M = \partial_\mu j^\mu_A} \quad \int \epsilon_A \partial_\mu j^\mu_A \propto \int_{M_{2n}} \cancel{\mathrm{Tr}\, F \wedge F \wedge \cdots \wedge F}$$

For fermions in rep R of group G and coupled to gravity the chiral anomaly is

$$\hat{A}(R)\, ch(F)\Big|_{2n}$$

$ch(F) = \mathrm{Tr}_R\, e^{iF/2\pi}$ - Chern character

$\hat{A}(R)$ = A roof genus ~~constructed in term of~~
$\sim 1 - (\;)\,\mathrm{Tr}\, R \wedge R + \cdots$

these are objects called characteristic classes which when integrated give topological invariants of vector bundles.

One can also have chiral bosonic fields which can contribute to the anomaly

$d=2$ - chiral bosons $d\phi = \pm * d\phi$

$d=6$ - chiral tensor fields B_2, $H_3 = dB_2$

$H_3 = \pm * H_3$,

Famously, in $d=10$ w/ $N=1$ susy one can cancel gauge and gravitational anomalies only for $G = E_8 \times E_8$, $Spin(32)/\mathbb{Z}_2$ (2 other cases)

Anomalies w/ Defects and Boundaries.

In QH physics there is an effective description using CS theory

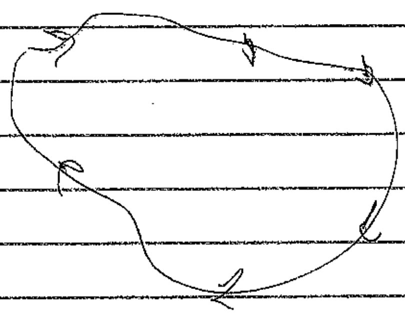
$$\int_{M_3} \omega_3(A) \propto \int_{M_3} A \wedge dA \quad \text{for } U(1) \text{ gauge theory}$$

Gauge variation $\delta_\lambda \int_{M_3} A \wedge dA = \int_{M_3} d\lambda \wedge dA$

$$= \int_{M_3} d(\lambda \wedge dA) = \int_{\partial M_3} \lambda \wedge dA \quad \text{Stokes}$$

To have a gauge invariant description there must be another contribution to the anomaly localized on ∂M_3

$\Rightarrow$ chiral edge states of QHE



In general there is a mathematical relation

$$\omega_{2n} = d\,\omega_{2n-1} \quad ; \quad \delta_\Lambda \omega_{2n-1} = d\,\omega_{2n-2}(\Lambda)$$

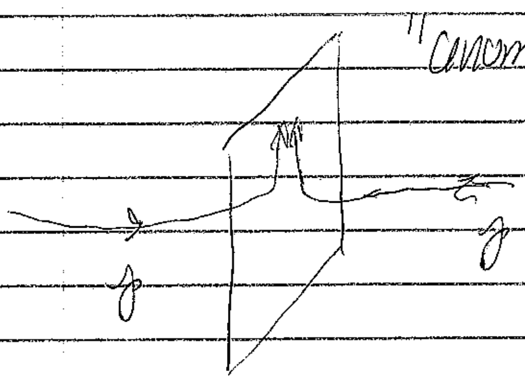
ω_{2n}: chiral anomaly in $2n$

ω_{2n-1}: CS terms in $2n-1$ dim

$\omega_{2n-2}(\Lambda)$: gauge/grav anomaly in $2n-2$ dimensions

i.e. $F \wedge F = d\omega_3 \qquad \delta\omega_3 = d(\Lambda\, dA)$ for $U(1)$ in $d=4$.

and this can be manifested physically for certain kinds of defects w/ localized chiral z.m. and hence anomalies in low-energy effective description.

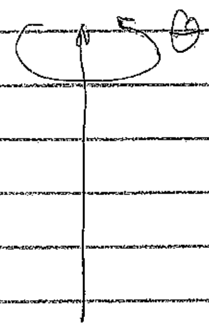


"anomaly inflow" w/ C. Callan

Bulk theory w/ CS couplings involving fields carrying charge of defect

defect w/ chiral z.m. ⇒ anomaly

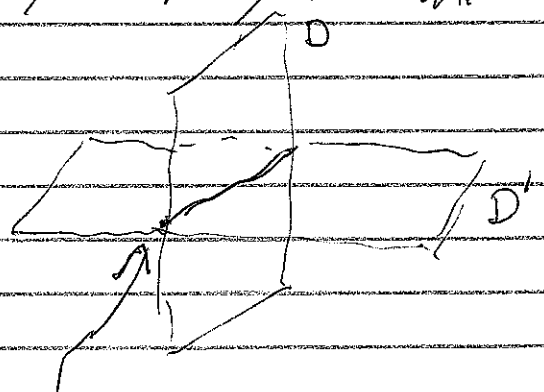
e.g. in theories w/ "axion strings" XXV



$$\int \Theta F \wedge F \sim \int d\theta\, \omega_3$$

$$d^2\theta \sim \delta^2(x)\, dx\, dy$$

Has many implications in D-brane physics where a class of couplings can be determined by anomaly inflow from intersecting D-branes



w/ Green + Moore

Also play an important role in M-theory - anomalies of M5-brane

Chiral z.m. on intersection

Condensed matter applications

I want to learn but a quick search shows papers involving anomalies and

QHE
Topological insulators
Weyl metals
chiral magnetic effect
hydrodynamics w/ anomalous currents.

In general the response ~~fam~~ to EM and gravitational fields of many systems w/o parity invariance are often dictated by chiral, gauge and gravitational anomalies.