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Color Superconductivity

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Overview

- Color superconductivity is predicted phase of QCD at ultra high density & low temperature
  - in particular for this talk will take $T=0$ & $\mu \gg m_s$

    $\Rightarrow$ treat u,d,s as massless to excellent approximation.

    symmetry $SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_B$

← worth mentioning decoupled in this region since strange mass $\sim \Delta_{QCD}$

as introduce $m_s \neq 0$ will have less symmetric ground state but whatever.

- Idea along lines of normal superconductivity

  $\exists$ attractive interaction among quarks

  $\Rightarrow$ formation of diquark bound states
  condensate.

  - vev breaks symmetry
  - argue to diagonal subgroup

    $SU(3)_D$ - simultaneous global flavor & color transformations.

  - called "color-flavor locked" or CFL phase

Outline:
- argue for CFL state
- Consequences
- Quantitative(ish) results for gap formation
  - not really reliable since sensitive to choice of form factor, etc...

Mainly follow Alford Rajagopal Wilczek (1999)

Color-Flavor-Locked State

- really educated guesswork as to what's energetically favorable.
- we know in this regime, quarks exert weak attraction

$\Rightarrow$ expect diquark condensate

$$\langle q^i_{\alpha a} q^j_{\beta b} \rangle \quad \langle \bar{q}^i_{\alpha \dot{a}} \bar{q}^j_{\beta \dot{b}} \rangle$$

i = flavor
α = color
$\dot{a}$ = spin.

Condensate should be Spin 0

- Argue spin 1 component will not compete w/ spin 0 component (anisotropy $\Rightarrow$ entire fermi surface cannot contribute coherently)

Parity preserving

- Simply don't see any sign of parity violation, even when include instanton effects (see 1998 paper by same authors)

P inv. state energetically favored in variational calc w/ t' Hooft vertex

so pick parity invariant state.

$$\langle q^i_a q^j_\beta \rangle = -\langle \bar{q}^i_a \bar{q}^j_\beta \rangle \quad \circledast'$$

- color antisymmetric channel is the attractive channel

$\Rightarrow$ flavor antisymmetric as well

so expect something like

$$\langle q^i_\alpha q^j_\beta \rangle = \kappa \epsilon^{ijI} \epsilon_{\alpha\beta I}$$

- turns out κ cannot solve self-consistent gap eqn & so need more general ansatz

$$\boxed{\langle q^i_\alpha q^j_\beta \rangle = \kappa_1 \delta^i_\alpha \delta^j_\beta + \kappa_2 \delta^i_\beta \delta^j_\alpha} \quad \circledast$$

- State not invariant under individual color & chiral flavor rotation or $U(1)_B$, but is under the diagonal subgroup

$\circledast$ locks $SU(3)_c = SU(3)_L$ & then $\circledast'$ $SU(3)_c$ to $SU(3)_R$.

- hence called color flavor locked phase.

only really robust predictions of the paper.

Immediate Consequences - Single particle excitations

- will see that κ_1 & κ_2 ~~[illegible]~~ gaps for the 9 types of quasiparticles
  - uniform over fermi surface so no low energy single particle excitations.
- Higg's mechanism.
  - Since color locked to flavor, no local symmetries remain
    $\Rightarrow$ all gluons aquire mass.
- 8 broken chiral generators, 1 broken $U(1)$ generator.
  - $SU(3)$ octet & singlet of Nambu-Goldstone bosons
- $U(1)_A \xrightarrow[\text{broken}]{} Z_6$ by instantons further broken to Z_2 by the condensate.
- Quark masses would break flavor symmetry & give the $SU(3)$ octet small masses & diminish symmetry of condensate.
  - $U(1)_B$ still massless.

Determining the Gap

- Single gluon exchange $\Rightarrow$ attractive interaction.

$\therefore$ use model hamitonian based on this color structure

$$H_I = 2K \int d^3x\, \mathcal{F}\left(\delta^\alpha_\delta \delta^\gamma_\beta - \frac{1}{3}\delta^\alpha_\beta \delta^\gamma_\delta\right) \bar{\psi}^i_\alpha \bar{\psi}^j_\gamma \psi^\beta_i \psi^\delta_j$$

$\mathcal{F}$ symbolizes momentum dependent form factor $F(p)$ for each leg of interaction to mock up asymptotic freedom.

will take $$F(p) = \left(1 + \exp\left(\frac{p-\Lambda}{w}\right)\right)^{-1}$$

- will just kind of pick Λ & w, pretty arbitrary, so this calculation cannot be truly trusted at a quantitative level, but will hopefully give insight into what's happening.

- Now as in usual BCS theory, make mean field approx

$$\bar{\Psi}^i_\alpha \bar{\Psi}^j_\beta \simeq \underbrace{\frac{3}{4K} P^{ij}_{\alpha\gamma}}_{\text{vev.}} + \underbrace{\delta b^{ij}_{\alpha\beta}}_{\text{small.}}$$

We parameterize the VEV by

$$P^{ij}_{\delta\gamma} = \frac{1}{3}(\Delta_8 + \tfrac{1}{8}\Delta_1)\delta^i_\alpha \delta^j_\gamma + \frac{1}{8}\Delta_1 \delta^i_\gamma \delta^j_\alpha$$ since Δ_1 & Δ_8 will turn out to be gaps.

- After this approx the total hamiltonian becomes

$$H = \int d^3x\, \bar{\Psi}(\not{\partial} - \mu\gamma_0)\Psi + \frac{1}{2}\int d^3x\, \bar{\Psi}\, \Phi^{ij}_{\beta\delta} \Psi^\beta_i \Psi^\delta_j + c.c.$$

$$\text{where } \Phi^{ij}_{\beta\delta} = \Delta_8 \delta^i_\delta \delta^j_\beta + \frac{1}{3}(\Delta_1 - \Delta_8)\delta^i_\beta \delta^j_\delta$$

Quadratic form in $\binom{\Psi}{\bar{\Psi}}$, so can diagonalize to find quasiparticle spectrum

- First decompose into

$$\Psi_\rho = \frac{1}{\sqrt{V}} \sum_{\vec{k}} \begin{pmatrix} -\sin\frac{\theta}{2} e^{-i\phi} \\ \cos\frac{\theta}{2} \end{pmatrix} \left(a_{\rho\vec{k}} e^{-ik\cdot x} + b^\dagger_{\rho\vec{k}} e^{ik\cdot x} \right)$$

(θ, ϕ) direction of $\vec{k}$

$\rho = 1, \ldots, 9$ is color-flavor index.

Diagonalizing Φ in color-flavor space we find

$$H = \sum_{\rho, k > \mu} (k - \mu) a^\dagger_{\rho\vec{k}} a_{\rho\vec{k}} + \sum_{\rho, k < \mu} (\mu - k) a^\dagger_{\rho\vec{k}} a_{\rho\vec{k}} + \sum_{\rho, \vec{k}} (k + \mu) b^\dagger_{\rho\vec{k}} b_{\rho\vec{k}}$$

$$+ \frac{1}{2} \sum_{\rho, \vec{k}} F(k)^2 \Phi_\rho e^{-i\phi(\vec{k})} \left(a_{\rho\vec{k}} a_{\rho-\vec{k}} + b^\dagger_{\rho\vec{k}} b^\dagger_{\rho-\vec{k}} \right) + c.c.$$

where $\Phi_1 = \Delta_1$ $\Phi_{2,\ldots,9} = \Delta_8$ are eigenvalues of Φ.

Can be diagonalized through some unenlightening algebra, to

$$H = \sum_{k, \rho} \sqrt{(k-\mu)^2 + F(k)^4 \Phi_\rho^2}\; y^\dagger_{\rho\vec{k}} y_{\rho\vec{k}} + \sqrt{(k+\mu)^2 + F(k)^4 \Phi_\rho^2}\; z^\dagger_{\rho\vec{k}} z_{\rho\vec{k}}.$$

So we see physical gaps of $F^2(k)\Delta_1$ & $F^2(k)\Delta_8$.

∘ As in BCS theory, Δ_1, Δ_8 must satisfy a self-consistency condition that can be used to solve for them.

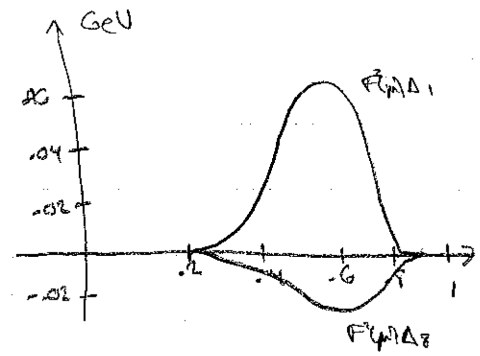
$$\langle \bar{\Psi}^i_\alpha \Psi^j_\beta \rangle = \frac{3}{4K} P^{ij}_{\alpha\beta}$$

plug in $\bar{\Psi}$ in terms of y & z & evaluate in $T=0$ state $|\Psi\rangle$ s.t. $y_{p\vec{k}}|\Psi\rangle = z_{p\vec{k}}|\Psi\rangle = 0$.

$$\Rightarrow \quad \Delta_8 + \frac{1}{4}\Delta_1 = \frac{4}{3}KG(\Delta_1) \qquad \frac{1}{8}\Delta_1 = \frac{4}{3}KG(\Delta_8)$$

$$G(\Delta) = -\frac{1}{2}\sum_k \left(\frac{F(k)^2\Delta}{\sqrt{(k-\mu)^2 + F(k)^4\Delta^2}} + \frac{F(k)^2\Delta}{\sqrt{(k+\mu)^2 + F(k)^4\Delta^2}} \right)$$

Numerical sol.: Set $K = \frac{1}{2}$ value s.t. zero density, Chiral Gap is .4 GeV ← true value (arbitrary)



$\Lambda = .8$ GeV.

$\omega = .05$ GeV.

we are overestimating if only use 1 interaction, so take half.