Michael Geracie

Generalized Anyon Models

Notes: Preskill's Quantum Computation Course.

- Main Idea: create an abstract model of anyons
  - encodes all algebraic information
  - but no dynamical information
    - interactions, bound states etc...
  - if can construct a quantum computer out of this, will be highly robust to decoherence

Plan: 1) Labels, fusion & splitting ← examples
2) R-matrices
3) Fusion matrices
4) Braid matrices & restrictions. ← examples.

Labels, Fusion

- most basic information is a list of particle types

$$L = \{a, b, c, \dots\}$$

called label set or superselection sector.

- need information that encodes how two particles might combine to form another
- specified by fusion rules

$$a \times b = \sum_c N^c_{ab}\, c$$

maps pairs of labels — to formal sums of labels

$2j+1$

- N^c_{ab} are non-negative integers
- physically N^c_{ab} represent no. of ways particles a + b may fuse to form particle c

Ex. two spin-$\frac{1}{2}$ particles

- two $\frac{1}{2}$'s can combine to form a 0 or 1.
- $\frac{1}{2}$ and a 1 can form a $\frac{1}{2}$ or a $\frac{3}{2}$, etc...

So label set is $L = \{0, \frac{1}{2}, 1, \frac{3}{2}, \dots\}$.

- fusion from representation theory

$$\tfrac{1}{2} \otimes \tfrac{1}{2} = 0 \oplus 1$$

(Q: see pg. 3)

So $\frac{1}{2} \times \frac{1}{2} = 0 + 1$

similarly for example octet rep of SU(3)

$$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27$$

$$\Rightarrow 8 \times 8 = 1 + 2 \cdot 8 + 10 + \overline{10} + 27.$$

Note:
- $\times$ is not direct product of vector spaces
- $+$ not direct sum
- only includes intrinsic algebraic properties of spins
  - nothing to do w/ forming bound states, bringing them close to eachother, etc...
- can always get fusion rules from representation theory, but there are some anyon models for which no picture in terms of reps exist
  - examples once have developed more machinery.

- N^c_{ab} must satisfy many combinatorial identities to be physically admissible

- will introduce them as we need them but lets get the two simplest ones out of the way

Commutivity

$a \times b = b \times a$

Associativity

$(a \times b) \times c = a \times (b \times c)$

- same charge whether you fuse a w/ b or b w/ a.
- " " " fuse a,b then c or b,c & then a.

Charge Conjugation & Vacuum

(After fusion spaces?)

- there is unique vacuum 0 s.t. $a \times 0 = a$ for all a.
  - corresponds to fusion w/ nothing.
  - in spin example, this is simply the spin 0 space 0.

(Q: so Ⓐ can't be right. It doesn't satisfy this! basically I need $0 \times j = j$ but I have $0 \times j = (2j+1) j$)

- ∃ a charge conjugation operator $C: L \to L$ s.t. $C^2 = 1$.
  - denote $Ca = \bar{a}$
  - demand $\bar{0} = 0$.
- will use later to relate spin & statistics.

(Do abelian case first & then nonabelian case).