Holographic EE

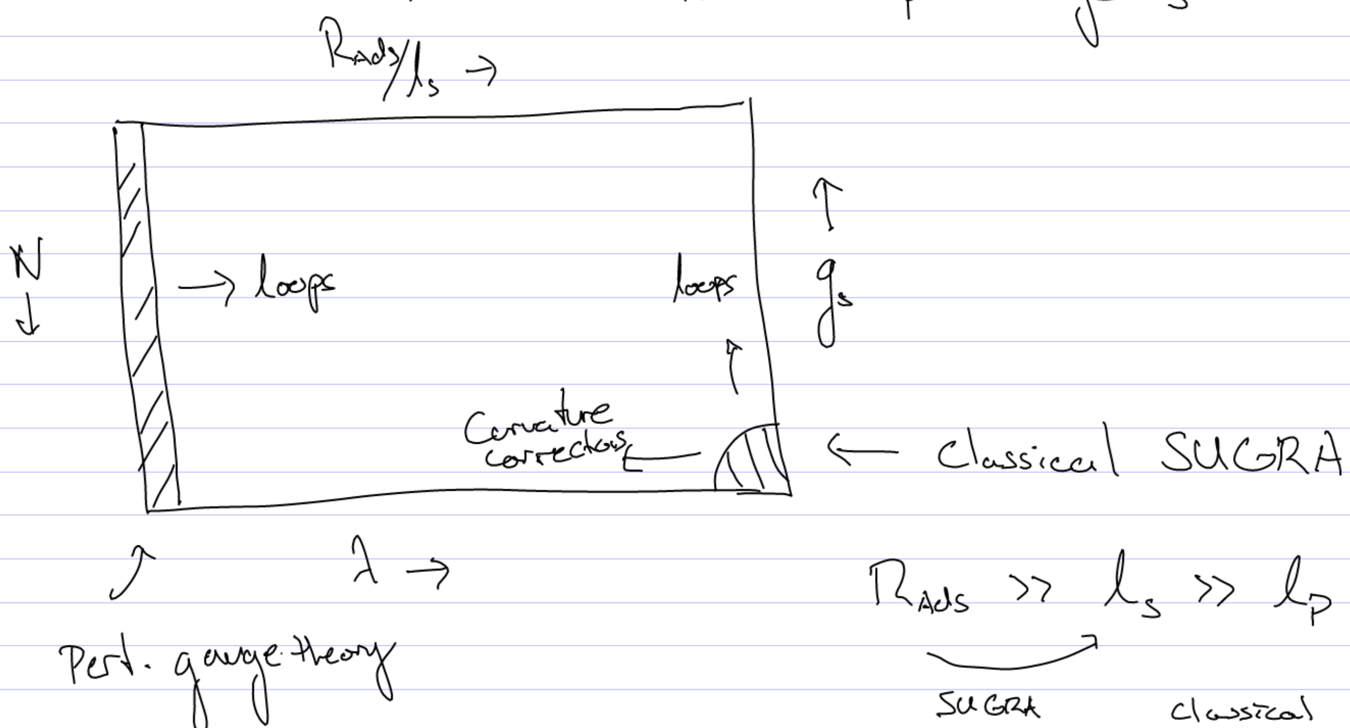
- Big topic, will only say a little

Outline:

- More on AdS/CFT (parameters, mixed states)
- EE in CFT/QFT
  ↳ some results quoted
- RT formula
- Checks
- Corrections

$\mathcal{N}=4$ SYM	$\longleftrightarrow$ IIB on $AdS_5 \times S_5$	
N	g_s	$g_s = g_{YM}^2 = \frac{\lambda}{N}$
$\lambda = g_{YM}^2 N$	$\frac{R_{AdS}}{l_s}$	$\left(\frac{R_{AdS}}{l_s}\right)^4 = \lambda$

Note: $G_N^{(10D)} \sim l_p^8 = g_s^2 l_s^8 = N^2 R_{AdS}^8$



$R_{AdS} \gg l_s \gg l_p$

SUGRA classical

Will use AdS_3/CFT_2, conceptually similar

$N^2 \rightarrow c$ of CFT_2

$\lambda \rightarrow g$ (marginal coupling)

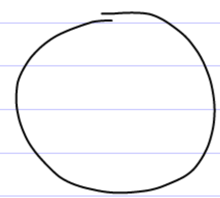
Correspondence: $Z_{string}[\phi \to \phi_0|_{\partial AdS}] = \langle \exp \int \phi_0 \mathcal{O} \rangle_{CFT}$

In CFT, State $\leftrightarrow$ operator

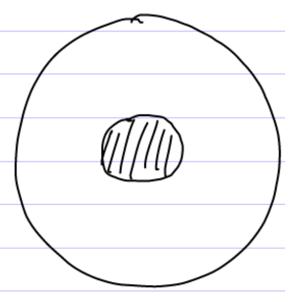
$\updownarrow$ Classical SUGRA

Solution to
Einstein w/ given bdry value

e.g. $|vac\rangle \longrightarrow$ pure AdS



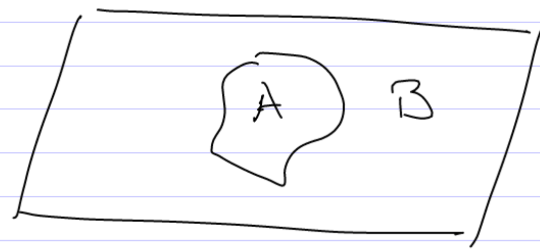
$|thermal(T)\rangle \longrightarrow$ AdS-BH w/ temp T



Some properties of EE in QFT

Recall: QFT on $\mathbb{R} \times M$

at $t = 0$



$A \cup B = M$

Given a dens. matrix ρ, (will assume ρ is dual to static spacetimes)

$\rho_A = \mathrm{tr}_B\, \rho$

$S_A = \mathrm{tr}_A (\rho_A \log \rho_A)$

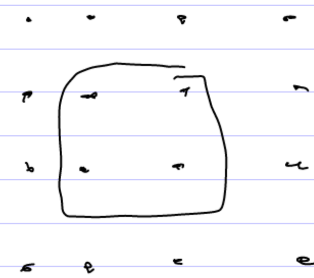
- $S_A^{(vac)} = S_B^{(vac)}$

- $S_{A \cup B} + S_{B \cup C} \geq S_{A \cup B \cup C} + S_B$

– "Area law"

S_A divergent. Regularize w/ UV cutoff $\epsilon \to 0$

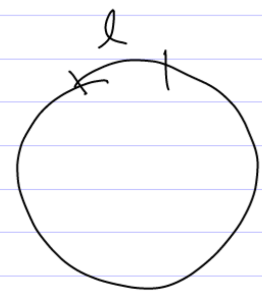
For $\rho = |\mathrm{vac}\rangle\langle \mathrm{vac}|$



$$S_A = \gamma \frac{\mathrm{Area}(\partial A)}{\epsilon^{d-1}} + \cdots$$

generally not universal

– In CFT_2: $M = S^1$ of length L

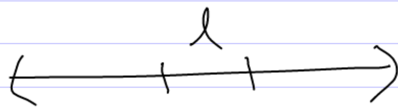


$$S_A^{(\mathrm{vac})} = \frac{c}{3} \log\left(\frac{L}{\pi\epsilon} \sin\left(\frac{\pi l}{L}\right)\right)$$

(note: symmetric under $l \to L-l$)

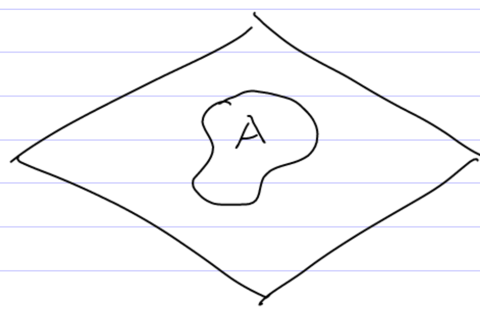
$$\xrightarrow[l \ll L]{} \frac{c}{3} \log\left(\frac{l}{\epsilon}\right)$$

— CFT_2 on $M = \mathbb{R}$ at $T = \frac{1}{\beta}$



$$S_A^{(\beta)} = \frac{c}{3} \log\left(\frac{\beta}{\pi \epsilon} \sinh\left(\frac{\pi l}{\beta}\right)\right)$$

— CFT_4 on $\mathbb{R}^3$ in vacuum:



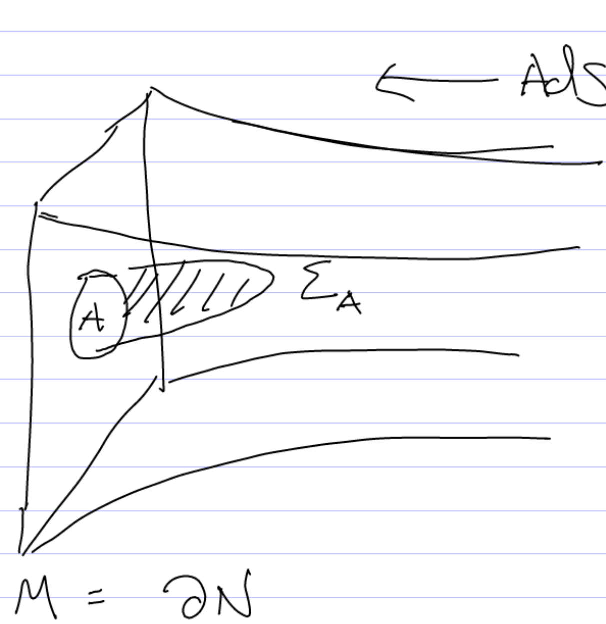
l is length scale of A

$$S_A^{(vac)} = \cdots + \gamma(a,c) \log\left(\frac{l}{\epsilon}\right) + \cdots$$

linear combination of a, c central charges

depends on geom. of ∂A

Ryu-Takayanagi formula:



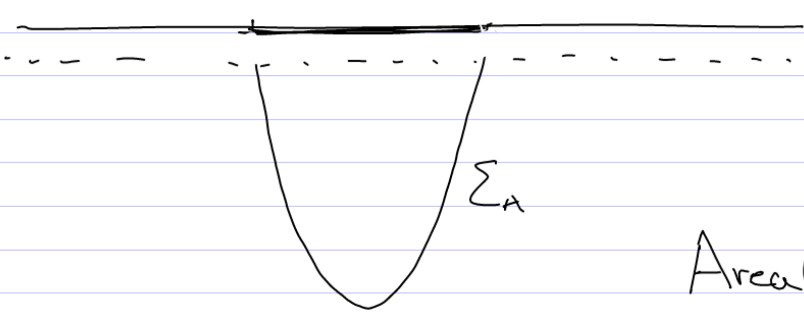
$$S_A^{(d+1)} = \frac{\text{Area}(\Sigma_A)}{4 G_N^{(d+2)}}$$

Σ_A is minimal area

$\downarrow$

extremal when bulk not static

$$ds^2_{AdS} = \frac{R^2_{AdS}}{z^2}\left(dz^2 - dt^2 + d\vec{x}^2\right)$$



M @ $z = 0$

M_ϵ @ $z = \epsilon$

$\text{Area}(\Sigma_A) \to \infty$ as $\epsilon \to 0$

$\downarrow$

$$\text{Area}(\partial A) \int_\epsilon dz \left(\frac{R_{AdS}}{z}\right)^d$$

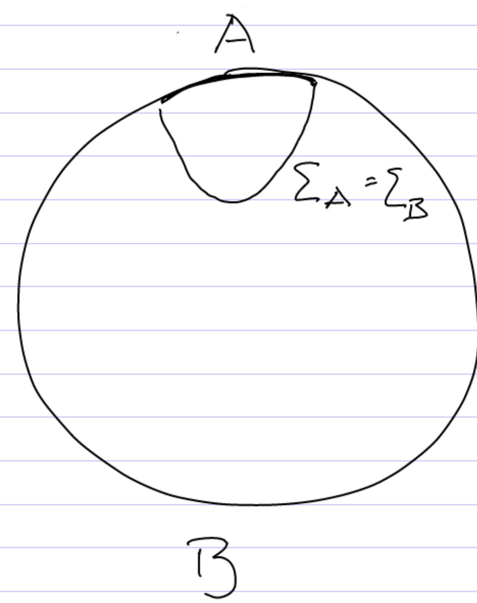
$$\sim \frac{\text{Area}(\partial A)\, R_{AdS}^d}{\epsilon^{d-1}}$$

$$\Rightarrow \quad S_A^{(d+1)} \sim \frac{\text{Area}(\partial A)}{\epsilon^{d-1}} \left(\frac{R_{AdS}^{d}}{G_N^{(d+2)}} \right)$$

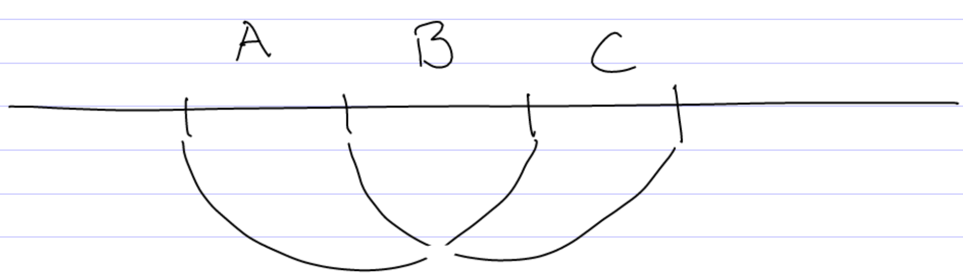
$\sim N^2$ in AdS_5

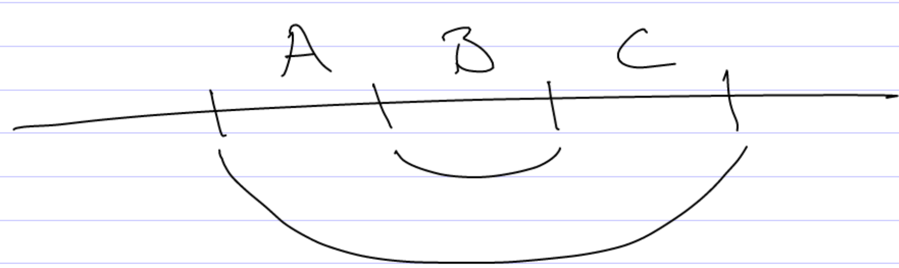
Now, drawings:

$S_A^{(vac)} = S_B^{(vac)}$



Strong subadditivity:

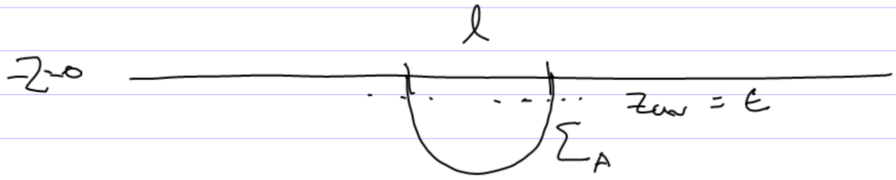




CFT_2 w/ infinite length, $|Vac\rangle \to ds^2 = \frac{R^2}{z^2}(-dt^2 + dz^2 + dx^2)$

Constant time slice in AdS_3

$$ds^2 = \frac{R^2}{z^2}(dz^2 + dx^2)$$



$$\text{Area}(\Sigma_A) = R \int d\lambda \, \frac{\sqrt{z'(\lambda)^2 + x'(\lambda)^2}}{z(\lambda)}$$

Minimizing: $(x, z) = \frac{\ell}{2}(\cos\lambda, \sin\lambda)$

$$\delta \leq \lambda \leq \pi - \delta$$

$$z_{uv} = \frac{\ell\delta}{2} = \epsilon$$

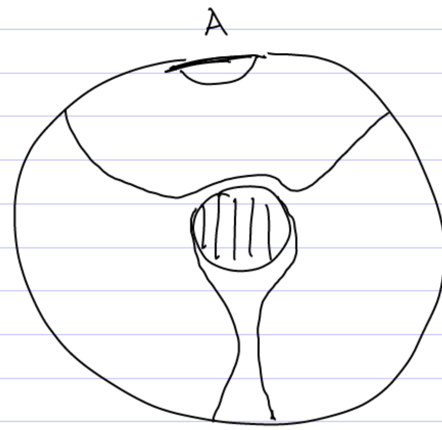
$$\text{Area}(\Sigma_A) = 2R \int_{\epsilon}^{\pi/2} \frac{d\lambda}{\sin\lambda} = 2R \log\left(\frac{\ell}{\epsilon}\right)$$

$$\Rightarrow S_A = \frac{R}{2G_N^{(3D)}} \log(\ell/\epsilon)$$

Miraculously, CFT_2 whose $|Vac\rangle \leftrightarrow AdS_3$

has $\frac{c}{3} = \frac{R}{2G_N^{(3)}}$

$$S_A = \frac{c}{3}\log\left(\frac{l}{\epsilon}\right)$$



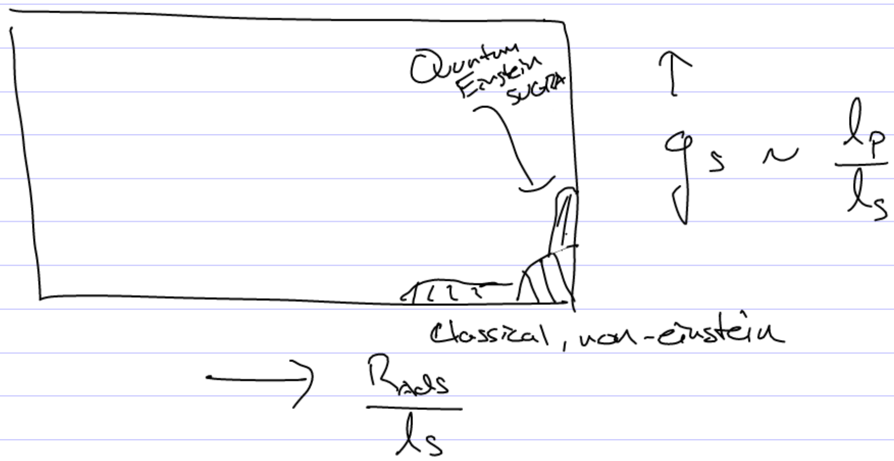
<u>finite T :</u>

$$S_A = \frac{c}{3}\log\left(\frac{\beta}{\pi\epsilon}\sinh\frac{\pi l}{\beta}\right)$$

$$\frac{l}{\beta}\to 0\ ,\quad S_A \to \frac{c}{3}\log\left(\frac{l}{\epsilon}\right)$$

$$\frac{l}{\beta}\to\infty\ ,\quad S_A \to \frac{c\pi l}{3}T \quad + \dots$$

Corrections?



EE in QG

Quantum: $S_A = \frac{\mathrm{Area}(\Sigma_A)}{4G_N} + S_\Omega$

$\partial\Omega = \Sigma_A$

Non-Einstein: $S_A = \underset{\Sigma_A}{\mathrm{ext}} \frac{\mathcal{F}(\Sigma_A)}{4G_N}$

Unknown (I think) what $\mathcal{F}(\Sigma_A)$ is

(not Wald)