1. Physical Phenomenon
2. Landau Levels. (Landau Gauge)
3. Disorder and Localization.
4. Landauer Approach.

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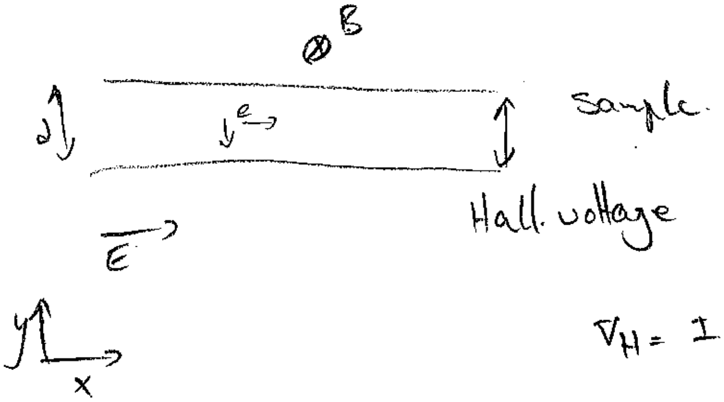
(1) **Physics** Classical Effect.



Hall voltage $\vec{v} \times B = V_H / d$

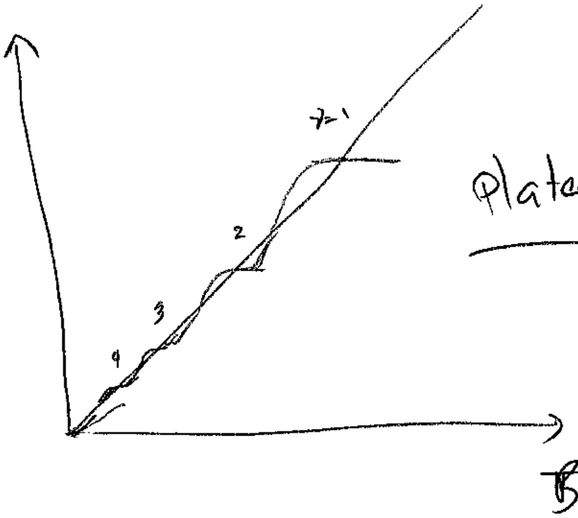
$$V_H = I R_H = d \rho e \vec{v} R_H$$

$$\Rightarrow vB = d\rho e v/d \, R_H \Rightarrow \boxed{R_H = \frac{B}{\rho e}}$$

put $\nu = \frac{\rho}{B/\Phi_0} = n \Rightarrow$ $\underbrace{R_H = \frac{h}{\nu e^2}}$

Classical Result

1980 von Klitzing:



Plateaus!

(2) Quantum Effect. (There is $\hbar$)

$$H = \frac{1}{2m}\left(p + \frac{eA}{c}\right)^2 \qquad \nabla \times A = B\hat{z} \quad \text{constant}$$

$\Rightarrow$ A is linear.

$\Rightarrow$ H is quadratic in coord and mom. $\rightarrow$ diagonalize

Gauge invce: $A \rightarrow A + d\alpha \qquad \psi \rightarrow e^{-ie\alpha}\psi$

leaves $H\psi = E\psi$ invariant.

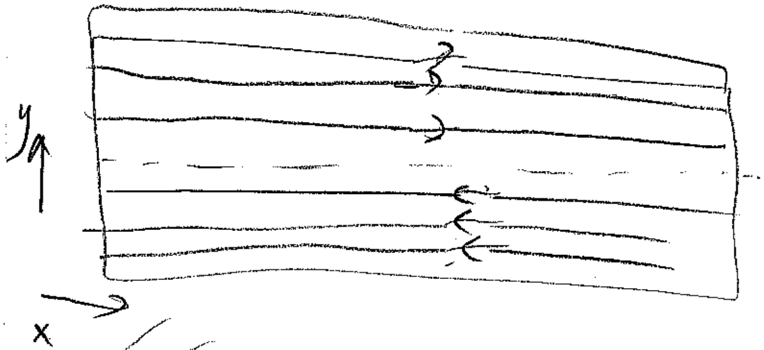
$\rightarrow$ Landau Gauge $A = B(-y, 0, 0)$ leaves

x translation invariance: $[H, p_x] = 0 \Rightarrow p_x = \hbar k_x$ is a good quantum number.

$$\rightarrow H_0 = \frac{1}{2m}\left(p_y^2 + (\hbar k_x - eBy)^2\right) \Rightarrow E = (n + \tfrac{1}{2})\hbar\omega_c$$

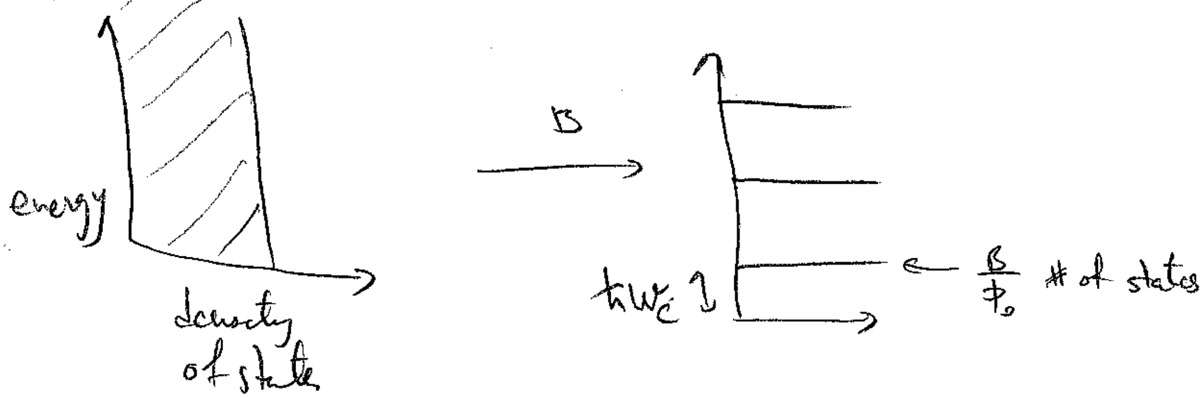
with $\boxed{y_{center} = l^2 k_x, \quad l^2 = \frac{\hbar c}{eB}}$ $\qquad \omega_c = \frac{eB}{m_e c}$

Note: All values of k_x have the same energy.



the $+k, -k$ states are separated.

$y = k_x l^2$ $\qquad$ $y > 0 \Rightarrow k > 0$



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* A full Landau level implies an excitation gap.

n full Landau levels $\Rightarrow$ $\nu = n$ $\Rightarrow$ $R_H = \frac{h}{ne^2}$ $\rightarrow$ Plateaus

We're done!

?

No. This is just a hint that we're on the right track. That full Landau levels correspond to plateaus.

③ Disorder & Localization

* Quick exercise. $\vec{B} = B_z \hat{z}$, $\vec{E} = 0$, $\vec{j} = 0$ → boost to frame with velocity $-v$ ⇒? electron velocity $= +v$ with $B = B\hat{z}$ $\vec{E} = -\vec{v} \times \vec{B}$ in leading order in v/c

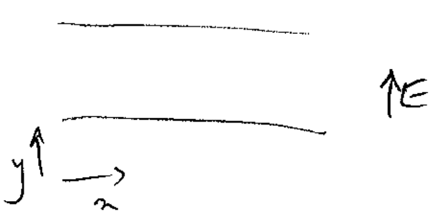
⇒ straight current w/o deflection needs $E = -v \times B$ exactly. No plateaus!

⇒ Something must break translation invariance so we can't use this argument any more.

→ $\boxed{\text{Disorder is crucial}}$

Effect of disorder.

$$H_0 \to H_0 + eEy$$



$$= \frac{1}{2m} p_y^2 + \frac{1}{2} m\omega_c^2 (y + k_x l^2)^2$$

~~The~~ $+ eEy$

Result $Y_k = -k_x l^2 - \frac{eE}{m\omega_c^2}$, $E_k = \frac{1}{2}\hbar\omega_c + eEY_k + \frac{1}{2} m\bar{v}^2$

with $\bar{v} = -cE/B = \frac{E \times B}{B^2}$ the usual electron drift.

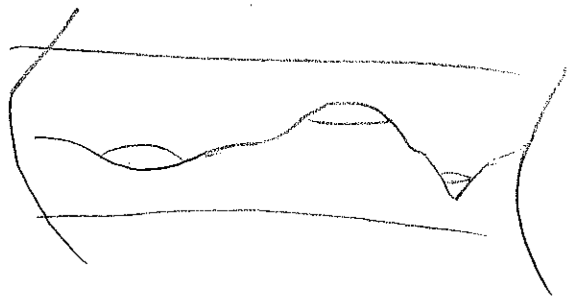
The electron drifts with velocity $\bar{v}$ $\langle j_y \rangle = -e\bar{v}$
or $\frac{1}{\hbar}\frac{\partial E_k}{\partial k} = \bar{v}$

→ In the presence of a potential, the guiding center of the electrons drifts <u>along equipotential lines.</u>

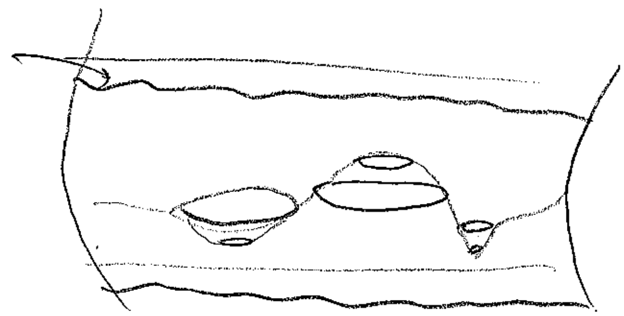
$E \times B \perp \nabla V \leftarrow$ potential.

→ Generalize

Potential with hills and troughs (Disorder.)



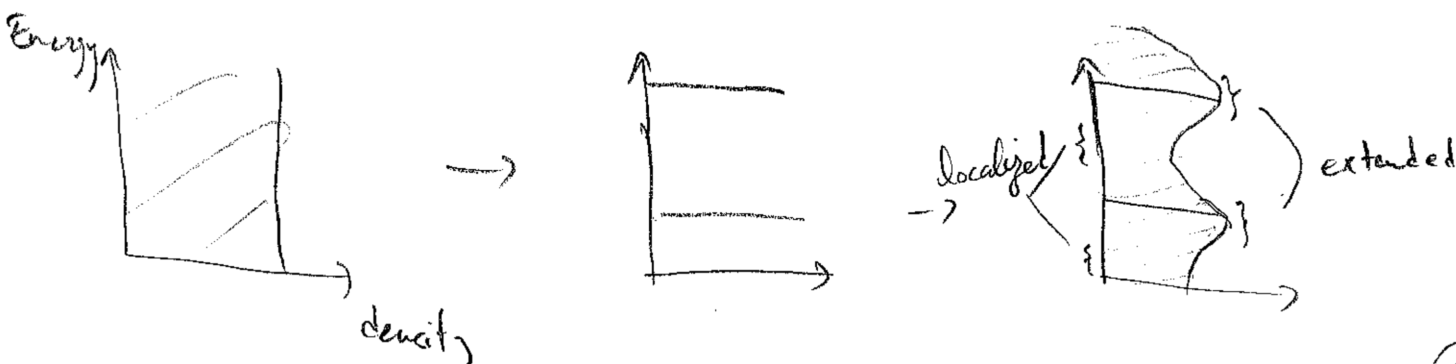
Draw the guiding center



localized states

Only when the phase matches when you go around the circle.

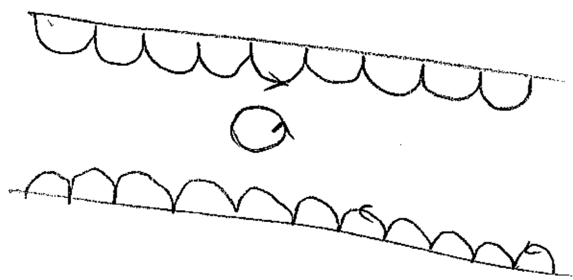
<u>Anderson Localization</u>



=> Status: Localized states in the bulk of the sample and extended states on edges. (Edge states.)

The edge states: Semiclassical interpretation:

Skipping Orbits.



* Localized States: Have to have area as multiple of Φ_0

Edge states form a continuum because they dont enclose any areas. (They resemble the unperturbed states most)

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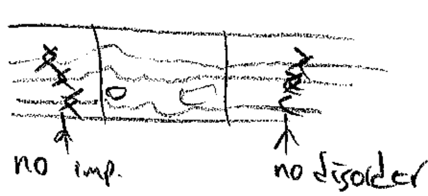
→ Intuitive Explanation: ② Right and left movers are spatially separated → Chance of backscattering is exponentially suppressed if sample width >> l_B. => $\rho_{xx} = 0$

=> If attach ideal ~~leads~~ to both sides:



$I = \frac{ne^2}{h} V_H$ ⊗ This current is the same because of current cont.

(*) The voltage in the ideal leads is the same as in the sample. ⇒ Quantization.

① Start with no impurity full Landau level. wave function
→ Add weak disorder → No mixing between Landau levels.
The disorder doesn't change the states' properties ⇒ The hall resistance doesn't change.

Now, if we add electrons (increase ν filling fraction)
The localized states get occupied and they don't contribute to transport (Mobility gap).

Landauer

Remember $Y_k = k\ell^2$

$$\text{derivation of } I = \frac{-e}{L_y}\int_{-\infty}^{\infty} dk \frac{L_y}{2\pi} \frac{1}{\hbar}\frac{\partial E_k}{\partial k} n_k$$

$$= \frac{-e}{h}\int_{\mu_{up}}^{\mu_{down}} dE = -\frac{e^2}{h} V_H \times \nu$$

$V_H = e\Delta\mu$, $\nu \to 1$ for each full Landau level.