Problem Set 4

Physics 221

Due October 29

Some abbreviations: B - Boas.

1. B. p.160 #43 (check that the eigenvectors are orthogonal) & #35 & #48 & #57 & #61

2. B. p.136 #16

3. Show that any anti-Hermitian matrix A satisfying $A^\dagger = -A$ can be diagonalized over the complex numbers (i.e. using a complex similarity transformation). What can you say about the eigenvalues of A? Suppose now that A is real. Can A be diagonalized over the real numbers (i.e. using a real similarity transformation)? If you think not, provide a counter example. If you think so, provide a proof.

4. The idea of Hermitian also applies to operators on functions as well as to finite-dimensional matrices. So let's consider continuous functions on the interval $[0, 2\pi]$. We defined an inner product between any two functions:

$$(f, g) = \int_0^{2\pi} dx\, f(x)^* g(x).$$

We need to specify boundary conditions on the functions that we want to consider. Let's take periodic functions satisfying

$$f(0) = f(2\pi).$$

(i) Show that the momentum operator $p = -i\frac{\partial}{\partial x}$ is Hermitian with respect to this inner product (recall the general definition of self-adjoint or Hermitian from lecture). In other words, show that $p^\dagger = p$.

(ii) What happens if we change the boundary condition so the functions are anti-periodic. Is p still Hermitian?

(iii) Find the most general boundary condition with respect to which p is Hermitian.

(iv) Lastly, show that the Hamiltonian $H = p^2$ is also Hermitian assuming periodic boundary conditions once again. Up to a minus sign, this operator is the Laplacian in one-dimension which you have seen in electrostatics.