Problem Set 7

Physics 221

Due December 3

Some abbreviations: B - Boas.

1. Find all the eigenfunctions and eigenvalues for the operator

$$-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}$$

acting on $\phi(x, y, z)$ in a square box with each side of length L. To do this, I want you to use the method of separation of variables which is perfect for this kind of operator. Take a candidate eigenfunction of the form

$$\phi(x, y, z) = X(x)Y(y)Z(z)$$

where $X(x)$ is just a function of x but not of (y, z) etc. Using this ansatz, you should be able to construct all the eigenfunctions. Lastly, take the boundary conditions:

$$\phi(0, y, z) = \phi(L, y, z) = 0,$$

$$\partial_y \phi(x, 0, z) = \partial_y \phi(x, L, z) = 0,$$

$$\phi(x, y, 0) = \phi(x, y, L) = 0.$$

2. Find the Green's function for the system in question **1** by representing the Green's function as a sum over eigenfunctions. The Green's function depends on $\vec{x}$ and $\vec{x}'$. As $\vec{x} \to \vec{x}'$, the boundary conditions should no longer be important. Can you show that in this limit the Green's function looks like the potential around a point charge?

3. Show that $\delta(ax) = \frac{\delta(x)}{|a|}$.

4. B. p.384 #8 & #21 & #27 & #35; for the first two problems, you can either use the book conventions for the definition of the Fourier transform or the lecture conventions.

5. Another useful transform like the Fourier transform is the Laplace transform. Read B. section 8, p.437 and solve B. p.438 #1. There is a typo in the text which should read: "Differentiate repeatedly with respect to a ..." (and not p). This will give you the basics of the Laplace transform.