1. Understanding the spectrum for simple potentials is very important. Consider an electron of mass $m$ in one dimension subject to a potential

$$V(x) = g\delta(x)$$

where $g$ is a constant and $\delta(x)$ is the Dirac delta-function. This system approximates a highly localized potential around the point $x = 0$.

(i) For what sign(s) of $g$ is there a quantum mechanical bound state?

(ii) Please find a relation between the bound state energy $E$ and $g$.

(iii) Now consider the double well potential

$$V(x) = g\delta(x + a) + g\delta(x - a).$$

Again find a relation between $E$, $g$ and $a$ for any bound states. What happens to the energy $E$ of these states as $a \to 0$?

2. This problem is inspired by a discussion after a lecture from years past. The current more tractable form of the problem was proposed by Rhys Povey, who solved the original much less tractable version of the problem. For a repulsive square well potential of width $a$ and height $V_0$, we saw in lecture that perfect transmission is possible as long as $\sin(ka) = 0$ where $k$ and the energy of the particle are related by

$$\hbar k = \sqrt{2m(E - V_0)}.$$

Suppose for an experiment, you want to engineer a potential that allows perfect transmission of particles with energies $E_1, E_2$, or $E_3$.

(i) First let’s explore a warm up case. Given two incoming particles with arbitrary but given energies $E_1$ and $E_2$, design a square wall potential (width and height) that permits perfect transmission of both particles.

(ii) Using two square walls of the type above, it is possible to allow a third arbitrary energy $E_3$ to also perfectly transmit through the system. Find the spacing between the two walls that allows perfect transmission of three given energies $E_1, E_2$ and $E_3$. You may assume, without loss of generality, that $E_3$ is greater than $E_1$ and $E_2$. 

Some abbreviations: S - Shankar.