The Hierarchy Problem

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1 Introduction

1.1 The Hierarchy Problem

The Hierarchy Problem tends to be the overarching name used by physicists to refer to a couple of different things, the first of which is the difference in scales between the mass of the electroweak gauge bosons/Higgs and the Planck mass. The calculation of the Higgs mass using the Quantum Field Theory of the Standard Model shows that it receives contributions from all energy scales, all the way up to the highest energy scale λ at which the Standard Model is valid. The most obvious choice is thus the Plank mass. This difference of many orders of magnitude between $M_{W,Z,H} \approx 100\, GeV$ and $M_{Planck} \approx 10^{19}\, GeV$ is the hierarchy referred to in the name of "The Hierarchy Problem". This vast difference in scale, on the order of $\approx 10^{17}\, GeV$, along with the naturalness problem discussed in the next section has convinced many physicists that there must be physics beyond the Standard Model, or that at the very least we do not understand fully explanation behind the hierarchy, and the fine tuning discussed in the next section.

1.2 Naturalness and the Higgs Mass

With the experimental result of the Higgs mass at $M_h = 125\,GeV$ this leaves a very big question of how one goes from the QFT expectation for the Higgs mass to the experimental result. The mass term of the Higgs in the Standard Model

$$m^2 H^\dagger H \tag{1}$$

is invariant under gauge or global symmetry on H, which results in the Higgs mass parameter being open to alterations by radiative corrections. Thus, the Higgs mass is modified by corrective terms from every scale with which it interacts; terms that are proportional to those scales. For the Standard Model, as discussed above, this scale can go all the way to $M_{Planck} \approx 10^{19}\,GeV$, and so the QFT expectation for the Higgs mass is much higher than the experimental result.

Currently these two values are brought together through a numerical cancellation of terms that results in the Higgs mass being reduced to its proper experimental values. But relying on a numerical cancellation is uncomfortable (dare I say, *unnatural*) for many physicists. This has spurred a number of alternative theories which solve the problem of the disparity of expected and experimental Higgs mass values without requiring fine tuning in the terms of the calculation.

2 The Anthropic Principle [1]

Before discussing proposed theoretical solutions to the hierarchy and naturalness problems, let's delve into the realm of anthropic explanations to the hierarchy problem. These principles all rely on the concept that for observation of the universe to occur, the current values of things like the Higgs mass, cosmological constant, and strong interaction are vital to the existence of ”life” or structure in the universe. A similar phrasing of the Anthropic Principle is that the current fine-tuned values are ”just how it is”, these physical parameters must be these numbers to support the existence of humans. The argument is also made that the anthropic principle makes more sense in the context of the multiverse theories. Thus, however rare it is for a parameter to have a certain value, within the multiverse there is a universe where it exists. Support of this idea seems to be fairly divisive among the physics theory community. It has also been applied to other areas such as the cosmological constant argument from Weinberg [3]. Anthropic theories are unable to be experimentally tested and thus will remain a somewhat nebulous concept until such a time arises.

3 Supersymmetry

Supersymmetry (SUSY) has long been touted as an elegant and beautiful theory that fixes the naturalness problem of the Higgs mass in the Standard Model. This happens through so called ”miraculous cancellation” done by the SUSY partners in the calculation of the Higgs mass and other similar observables.[2]

This can be demonstrated by first finding a Lagrangian invariant under a SUSY transformation. The simplest option is using chiral superfields, the Kahler Potential ($K(\Phi, \Phi^\dagger)$), and the superpotential $W(\Phi)$ defined as:

$$K(\Phi, \Phi^{\dagger}) = \Phi^{\dagger}\Phi \tag{2}$$

$$W(\Phi) = \frac{m}{2}\Phi^2 + \frac{g}{3}\Phi^3. \tag{3}$$

The complex scalar ψ and the 4 spinor Φ are defined as

$$\phi = \frac{(A + iB)}{\sqrt{2}} \tag{4}$$

$$\Psi = (\psi, \bar{\psi}). \tag{5}$$

which gives the Lagrangian:

$$\mathcal{L} = \partial^{\mu}\phi^{*}\partial_{\mu}\phi + i\bar{\psi}\bar{\sigma}^{\mu}\partial_{\mu}\psi - |m\phi + g\phi^2|^2 - (\frac{m}{2} + g\phi)\psi\psi \tag{6}$$

$$= \frac{1}{2}\partial^{\mu}A\partial_{\mu}A - \frac{1}{2}m^2A^2 + \frac{1}{2}\partial^{\mu}B\partial_{\mu}B - \frac{1}{2}m^2B^2 + \frac{1}{2}\bar{\Psi}(i\not{\partial} - m)\Psi \tag{7}$$

$$- \frac{mg}{\sqrt{2}}A(A^2 + B^2) - \frac{g^2}{4}(A^4 + B^4 + 2A^2B^2) - \frac{g}{\sqrt{2}}\bar{\Psi}(A - iB\gamma^5)\Psi \tag{8}$$

As an example the 1-loop corrections to the mass of the scalar A can be computed from the Feynman diagrams in **Figure 1**.

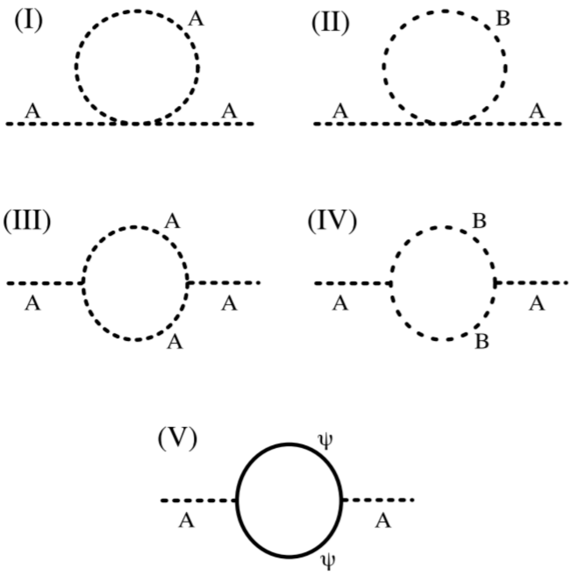

Figure 1: One loop diagrams correcting the mass of the scalar A[2]

Using Feynman rules, these loop diagrams give the respective contributions of

$$(I) = 3g^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{k^2 - m^2} \tag{9}$$

$$(II) = g^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{k^2 - m^2} \tag{10}$$

$$(III) = 3g^2 m^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{(k^2 - m^2)((k-p)^2 - m^2)} \tag{11}$$

$$(IV) = g^2 m^2 \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{(k^2 - m^2)((k-p)^2 - m^2)} \tag{12}$$

$$(V) = -2g^2 (\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{k^2 - m^2} + \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{(k-p)^2 - m^2} \tag{13}$$

$$+ \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{(k^2 - m^2)((k-p)^2 - m^2)}) \tag{14}$$

The signs of the corrective terms resulting from the integrals corresponding to bosonic diagrams (I - IV) are opposite the sign of those corresponding to the fermionic diagram (V) thus allowing the UV divergent pieces to cancel resulting in a total mass correction term of:

$$2g^2 \{ \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{k^2 - m^2} - \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{(k-p)^2 - m^2} + \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{(k^2 - m^2)((k-p)^2 - m^2)}) \}. \tag{15}$$

Thus the cutoff energy scale enters only logarithmically:

$$\int_{\Lambda} \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{1}{(k^2-m^2)((k-p)^2-m^2)}) \approx \int_0^{\Lambda} \frac{2\pi^2 k^3 \mathrm{d}k}{(2\pi)^4} \frac{1}{k^4} \sim \int_0^{\Lambda} \frac{\mathrm{d}k}{k} \sim ln(\Lambda), \tag{16}$$

instead of quadratically like the divergences of most non-supersymmetric theories: [2]

$$\int_{\Lambda} \frac{\mathrm{d}^4k}{(2\pi)^4} \frac{1}{k^2-m^2} \approx \int_0^{\Lambda} \frac{2\pi^2 k^3 \mathrm{d}k}{(2\pi)^4} \frac{1}{k^2} \sim \int_0^{\Lambda} k\, \mathrm{d}k \sim \Lambda^2. \tag{17}$$

A very similar process happens with the Higgs mass calculation with SUSY. The contributions from the additional fermionic and bosonic partners cancel the contributions from the SM particles, allowing the Higgs mass to remain low, as the experimental results have shown it to be [4].

This is a very elegant solution to the naturalness problem surrounding the Higgs which is one of the reasons why Supersymmetry is such a popular theory. Unfortunately, it remains undiscovered as yet at the LHC, and the higher energies reached without the discovery of a supersymmetric partner, the less viable the theory is for protecting the Higgs mass.

4 Little Higgs

The Little Higgs model introduces a new mechanism for electroweak symmetry breaking in which the Higgs is a pseudo Nambu-Goldstone boson. It's mass is protected from the type of large radiative modifications seen in Standard Model calculations by approximate nonlinearly realized global symmetries. The symmetries are explicitly broken, but softly enough that the Higgs mass term is $\approx ln(\Lambda)$ assuming that none of the terms in the Lagrangian break all the global symmetries protecting the Higgs mass. This leads to a few new particles ($\sim TeV$ scale) which cancel the leading quadratic terms in the Higgs mass (in a similar manner as discussed in the SUSY section above) [5].

The Little Higgs model was developed by starting with a SU(k) five dimensional gauge theory compactified on a circle of radius R [6]. The zero mode of the gauge potential decomposes into the SU(k) gauge bosons and a real massless scalar field, which acquires a 1-loop corrective term to the mass at low energies which quadratically diverges [6]. At energies higher than $\frac{1}{R}$ there are no contributions to the mass of the scalar due to the higher dimensional gauge invariance, resulting in a finite correction to the scalar mass [6]. The (originally massless but now) light scalar is a pseudo Nambu-Goldstone boson which the Higgs can be associated with [6]. Dimensional deconstruction which utilizes a non-linear sigma model shows that extra dimensions can be discarded leaving four dimensional electroweak symmetry breaking without the need for fine tuning [6].

Despite modifications to Little Higgs models, electroweak precision observables at the LHC constrain the symmetry breaking scale to be larger than 3 TeV at a 95% confidence level. After introducing a discrete symmetry T-parity this limit is lowered to around 1.8 TeV at 95% confidence.

5 Conclusion

The hierarchy problem continues to be an issue with the Standard Model. As time goes on and experimental evidence continues to not be found even as the energy scales probed increase; many of these theories will become less viable. Many people still hold out hope for Supersymmetry to save the day - and I agree it seems like a particularly elegant solution, as does Little Higgs. But at heart, I am an experimentalist. And despite the convincing nature of (some of) the theories described, I await future results. In the meantime, I look forward to contributing to searches like [] and to the construction of the HL-LHC.

References

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