Spontaneous Symmetry Breaking and the Standard Model

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June 8, 2019

1 Introduction

The discovery of the Higgs boson at the Large Hadron Collider in 2012 marked a major milestone for particle physics; the Higgs discovery completed the Standard Model (SM) of particle physics, which has been a remarkably successful theory under experimental tests. Prior to the discovery of the Higgs, however, the experimental measurements of non-zero particle mass and the theoretical structure of the SM were in disagreement and required some addition to the theory. The Higgs mechanism resolved this tension through the application of spontaneous symmetry breaking, which does not explicitly break symmetries but rather 'hides' them.

This paper reviews the role of spontaneous symmetry breaking (SSB) in the Standard Model, both in experimentally-confirmed contexts and in proposed models designed to resolve some outstanding issues with the SM. In Section 2, we give a brief review of how the Higgs mechanism induces electroweak symmetry breaking (EWSB), then focus on the specifics of the quark sector and the possible origins of neutrino mass. We then move on to grand unified theories (GUT) in Section 3, emphasizing the specific representations of the SM matter content in these gauge groups and the phenomenological considerations associated with each group.

2 The Higgs mechanism

Because the Standard Model is built on the gauge group of $SU(3) \times SU(2) \times U(1)$, the most natural way to build the theory is with massless fields. In fact, mass terms for fermionic fields explicitly break $SU(2)$ symmetry:

$$\mathcal{L}_{mass} \supset -m_f \bar{\psi}\psi = -m_f \left(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R\right). \tag{1}$$

Such terms are not invariant under $SU(2)$, as ψ_R transforms as a singlet under $SU(2)$ while ψ_L transforms as part of an $SU(2)$ doublet. Meanwhile, mass terms for the weak gauge bosons also break this gauge symmetry:

$$\begin{aligned}\mathcal{L}_{mass} &\supset \frac{1}{2}M_A^2 A_\mu A^\mu \\ &\to \frac{1}{2}M_A^2 \left(A_\mu - \frac{1}{e}\partial_\mu \alpha\right)\left(A^\mu - \frac{1}{e}\partial^\mu \alpha\right) \\ &\neq \frac{1}{2}M_A^2 A_\mu A^\mu\end{aligned} \tag{2}$$

where we have chosen to illustrate the issue with the photon mass term due to its slightly simpler transformation; the principle for the weak gauge bosons is the same. The fact that mass terms explicitly break the SM gauge symmetry creates a tension between the theory, which does not inherently allow mass terms, and experiment, which has measured non-zero masses for the $W^\pm$ and Z gauge bosons as well as for the fermion content of the SM. Spontaneous symmetry breaking via the Higgs mechanism introduces mass terms to the SM theory without explicitly breaking the gauge symmetries. In this section, we will give a brief overview of the Higgs mechanism breaking $SU(2)_L \times U(1)_Y$ to $U(1)_Q$, then focus on the specifics of the quark sector and the possible origins of neutrino mass.

2.1 Breaking of $SU(2) \times U(1)$

Here we present a brief review of the mechanism through which the Higgs field breaks the $SU(2)_L \times U(1)_Y$ group to $U(1)_Q$. Until otherwise noted, the content contained in the following discussions is a combination of material from Refs. [1, 2].

A complex $SU(2)$ doublet field is introduced with a kinetic term and potential of the form:

$$\mathcal{L}_{\text{Higgs}} = (D^\mu \Phi)^\dagger D_\mu \Phi - \mu^2 \Phi^\dagger \Phi - \lambda(\Phi^\dagger \Phi)^2. \tag{3}$$

When $\mu^2 < 0$, Φ obtains a non-zero vacuum expectation value (vev) of $v^2 = \frac{-\mu^2}{\lambda}$, around which one may expand by

$$\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix} \tag{4}$$

after applying a gauge transformation to the unitary gauge. The field $H(x)$, which describes variations from the vacuum, is the field associated with the SM Higgs boson. The kinetic term of Φ provides masses to the weak gauge bosons:

$$(D^\mu \Phi)^\dagger D_\mu \Phi = \left| \left(\partial_\mu - \frac{i}{2} g \tau^a W^a_\mu - \frac{i}{2} g' B_\mu \right) \Phi \right|^2 \tag{5}$$

$$= \frac{1}{2} \partial^\mu H \partial_\mu H + \frac{1}{8} \left(g^2 (v+H)^2 |W^1_\mu + i W^2_\mu|^2 + (v+H)^2 |g W^3_\mu - g' B_\mu|^2 \right) . \tag{6}$$

We define the $W^\pm$, Z, and A gauge bosons as

$$W^\pm = \frac{1}{\sqrt{2}} \left(W^1_\mu \mp i W^2_\mu \right) \qquad Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g W^3_\mu - g' B_\mu \right) \qquad A_\mu = \frac{1}{\sqrt{g^2 + g'^2}} \left(g' W^3_\mu + g B_\mu \right) \tag{7}$$

so that the weak gauge bosons obtain masses, while the photon is massless. In particular, this feature of the photon arises from the fact that the vev is in the neutral component of the $SU(2)$ doublet Φ and therefore does not break EM charge symmetry. Because the photon is the generator of $U(1)_Q$ and is defined as a sum of W^3_μ and B_μ, the EM charges of SM particles are given by the matrix $Q = T^3 + Y$, where T^3 is the third generator of $SU(2)$ in the appropriate representation and Y is the hypercharge of the particles under $U(1)_Y$.

2.2 The quark sector

The coupling of the Higgs doublet to quarks is given by the interaction terms

$$\mathcal{L}_{H,\text{quark}} = -\sqrt{2} (\lambda_d \bar{Q}_L \Phi d_R + \lambda_u \bar{Q}_L \tilde{\Phi} u_R + h.c.) \tag{8}$$

where $\tilde{\Phi} \equiv i\tau^2 \Phi^\dagger$ has hypercharge -1, as required to make the second term hypercharge-neutral. The presence of $i\tau^2$ causes $\tilde{\Phi}$ to transform under the fundamental representation of $SU(2)$, which is necessary as $\bar{Q}_L$ transforms under the anti-fundamental representation and u_R transforms as a singlet. Expanding about the vacuum, $\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & v + H \end{pmatrix}^T$, gives mass terms of the form

$$\begin{aligned}\mathcal{L}_{H,\text{quark}} &= -\lambda_d \begin{pmatrix} \bar{u}_L & \bar{d}_L \end{pmatrix} \begin{pmatrix} 0 \\ v+H \end{pmatrix} d_R - \lambda_u \begin{pmatrix} \bar{u}_L & \bar{d}_L \end{pmatrix} \begin{pmatrix} v+H \\ 0 \end{pmatrix} u_R - h.c. \\ &= -\lambda_d (v+H) \bar{d}_L d_R - \lambda_u (v+H) \bar{u}_L u_R - h.c. \\ &= -v\lambda_d \bar{d}_L d_R - v\lambda_u \bar{u}_L u_R - h.c. + (\text{interaction terms}). \end{aligned} \tag{9}$$

The matrices $\lambda_{d,u}$ are not necessarily diagonal, and must therefore be diagonalized in order to identify the quark masses and mass eigenstates. We write the $\lambda_{d,u}$ in terms of diagonal matrices as

$$\lambda_d = U_d \Lambda_d S_d^\dagger \qquad \lambda_u = U_u \Lambda_u S_u^\dagger. \tag{10}$$

With this diagonalization, the quark fields transform as

$$u_L \to U_u u_L \qquad u_R \to S_u u_R \tag{11}$$

$$d_L \to U_d d_L \qquad d_R \to S_d d_R \tag{12}$$

where U and S are unitary 3x3 matrices. Putting these replacements into the Lagrangian, the mass terms are now diagonalized:

$$\begin{aligned}\mathcal{L}_{H,\text{quark}} &\supset -v \left(\bar{d}_L U_d^\dagger U_d \Lambda_d S_d^\dagger S_d d_R^j + \bar{u}_L U_u^\dagger U_u \Lambda S_u^\dagger S_u u_R + h.c. \right) \\ &= -v \left(\bar{d}_L \Lambda_d d_R + \bar{u}_L \Lambda_u u_R \right). \end{aligned} \tag{13}$$

Since the matrices $\Lambda_{d,u}$ are diagonal, we may define $m_{u,d}^i = \Lambda_{u,d}^{ii}$, where i indicates the generation index, and write (with sum over i implicit)

$$\mathcal{L}_{\text{quark,mass}} = -v \left(m_d^i \bar{d}_L^i d_R^i + m_u^i \bar{u}_L^i u_R^i + h.c. \right) \tag{14}$$

We have now obtained mass terms for the quarks in terms of the mass eigenstates. However, unlike in the lepton sector, rewriting the rest of the Lagrangian in terms of the mass eigenstates does not leave the other terms invariant. In particular, we can see this in the charged weak current $J^{\pm}$, which couples to $W^{\pm}$:

$$\begin{aligned}
\frac{1}{2}(J^{\mu,+} + J^{\mu,-}) &= \bar{u}_L\gamma^\mu d_L + \bar{d}_L\gamma^\mu u_L \\
&= \bar{u}_L U_u^\dagger \gamma^\mu U_d d_L + \bar{d}_L U_d^\dagger \gamma^\mu U_u u_L \\
&= \bar{u}_L^i \gamma^\mu (U_u^\dagger U_d)^{ij} d_L^j + \bar{d}_L^i \gamma^\mu (U_d^\dagger U_u)^{ij} u_L^j \\
&\equiv \bar{u}_L^i \gamma^\mu V_{\mathrm{CKM}}^{ij} d_L^j + \bar{d}_L^i \gamma^\mu (V_{\mathrm{CKM}}^\dagger)^{ij} u_L^j.
\end{aligned} \tag{15}$$

The mixing matrix V_{CKM} is known as the Cabibbo-Kobayashi-Maskawa matrix, and can be written in terms of three angles $(\theta_1, \theta_2, \theta_3)$ and a CP-violating phase δ.

We thusly see that the diagonalization of the mass matrices in terms of mass eigenstates introduces couplings between different quark generations within the weak sector. This indicates that the weak eigenstates are combinations of the mass eigenstates. Thus, the charged weak current may mediate flavor changing interactions in the quark sector. This contrasts with the neutral weak currents, which do not introduce flavor-changing interactions in either the quark or lepton sector, as well as with the Higgs-quark couplings, which are diagonalized along with the mass terms.

2.3 Neutrino masses

Within the lepton sector, the Higgs interaction terms are given by

$$\begin{aligned}
\mathcal{L}_{H,\mathrm{lept}} &= -\sqrt{2}\left(\lambda_l \bar{L}_l \Phi R_l + \bar{R}_l \Phi^\dagger L_l\right) \\
&= -\lambda_l \left(\begin{pmatrix} \bar{\nu}_l & \bar{l}_L \end{pmatrix} \begin{pmatrix} 0 \\ v + H \end{pmatrix} l_R + \lambda_l \bar{l}_R \begin{pmatrix} 0 & v + H \end{pmatrix} \begin{pmatrix} \nu_l \\ l_L \end{pmatrix} \right) \\
&= -\lambda_l \left(v\bar{l}_L l_R + v\bar{l}_R l_L \right) + (\text{interaction terms}) \\
&= -v\lambda_l \bar{l} l + (\text{interaction terms})
\end{aligned} \tag{16}$$

where the subscript l denotes the particular lepton generation (e, μ, τ) and we have defined $L_l = \begin{pmatrix} \nu_l & l_L \end{pmatrix}^T$ and $R_l = l_R$. The Higgs therefore gives mass to the electron, muon, and tau leptons, but leaves the neutrinos massless. We note that in this case, the charged weak current does not mediate inter-generational mixing, as the massless neutrino eigenstates may be defined by their coupling to the relevant partner lepton eigenstates. However, neutrinos are known to have mass due to neutrino oscillations, which indicates that the mass and flavor neutrino eigenstates are different.

The mechanism through which neutrinos receive mass is still unknown; here we will review the two primary scenarios. In the first case, neutrinos are Dirac fermions which have a right-handed counterpart and a mass term similar to that of up-type quarks. In the second case, neutrinos are Majorana fermions with Majorana mass terms. In both scenarios, the neutrinos may obtain their mass through the Higgs mechanism.

2.3.1 Dirac neutrinos

The first possibility is that the neutrino is a Dirac fermion, similarly to the other fermions in the SM. In this case, there will be an independent right-handed component which we denote as $N^i \equiv \nu_R^i$. Then we can write a neutrino mass term similarly to the up-type quark mass terms:

$$\mathcal{L}_{mass,\nu} = \lambda_\nu^{ij} \bar{L}^i \tilde{\Phi} N^j + h.c. \tag{17}$$

As before, diagonalizing yields a mixing matrix amongst the generations and mass terms of the form

$$\mathcal{L}_{mass,\nu} = m_\nu^i (\bar{\nu}_L^i \nu_R^i + \bar{\nu}_R^i \nu_L^i). \tag{18}$$

Although this method successfully introduces neutrino masses to the SM Lagrangian, the Yukawa couplings alone (along with the fixed scale v) are responsible for defining the mass scale of the neutrinos. In this case, the neutrino Yukawa couplings must be on the order of $\mathcal{O}(10^{-12})$ to reproduce neutrino masses of $\mathcal{O}(10^{-1})$ eV. These values of the Yukawa couplings are many orders of magnitude smaller than those of the other fermionic SM particles. Additionally, the most general Lagrangian can contain a Majorana mass term for the right-handed neutrinos, as they transform as singlets under the entire $SU(3)\times SU(2)\times U(1)$ gauge group. These two considerations help motivate an analysis of the Majorana case.

2.3.2 Majorana neutrinos

Here we present material contained in Ref [3]; some of the development of the seesaw models may be found in Refs. [4, 5]. Because the neutrino is electrically neutral, it is possible that the neutrino is a Majorana fermion, where the neutrino is its own antiparticle. We may check that the neutrino is electrically neutral in the SM after EWSB by calculating the lepton charge matrix Q which we derived previously. In the case of a left-handed lepton doublet, the relevant $SU(2)$ generator T^3 is τ^3 of the fundamental representation, $\frac{1}{2}\sigma^3$:

$$QL = (\tau^3 + Y)L = \begin{pmatrix} \frac{1}{2} - \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} - \frac{1}{2} \end{pmatrix} \begin{pmatrix} \nu_L \\ l_L \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu_L \\ l_L \end{pmatrix} \tag{19}$$

where we have identified that the hypercharge must be $Y = -1/2$ in order to obtain the correct charge for the electron, muon, and tau. Thus the neutrino is indeed electrically neutral under the definition of Q obtained through the Higgs mechanism.

For a particle to be a Majorana fermion, the operator which creates and annihilates the particle must be the same. In this case, the charge-conjugate field $\nu^c(x) = C\bar{\nu}^T(x)$ is equal to the original field $\nu(x)$, and therefore the right-handed component is not an independent component of the field but rather the charge-conjugate of the left-handed component: $\nu_R = \nu_L^c$. Majorana mass terms are therefore given by

$$\mathcal{L}_{mass,\nu} = -\frac{1}{2} m_\nu \left(\bar{\nu}_L^c \nu_L + \bar{\nu}_L \nu_L^c \right). \tag{20}$$

As before, such mass terms explicitly break $SU(2)$ symmetry. One may obtain terms of this form through the Higgs mechanism by including the previous Higgs-neutrino interaction term as well as introducing Majorana mass terms for heavy right-handed neutrinos of the form

$$\mathcal{L}_{H,\text{lept}} \supset -\lambda_\nu^{i\alpha} \bar{L}^i \tilde{\Phi} N^\alpha - \frac{1}{2} M^{\alpha\beta} \bar{N}^\alpha (N^\beta)^c + h.c. \tag{21}$$

The generation indices on the left-handed and right-handed neutrinos are differentiated to indicate that the number of right-handed neutrino generations has not been specified, and may not be equal to 3. With these terms, the diagonalization of the resulting mass matrix after EWSB leads to Majorana mass terms for the left-handed neutrino. Explicitly, EWSB yields a Dirac mass matrix $m_D^{i\alpha}$, which combines with the Majorana mass matrix $M^{\alpha\beta}$ to give

$$\mathcal{L}_{H,\text{lept}} \supset -\frac{1}{2} \begin{pmatrix} (\bar{\nu}_L^i)^c & \bar{N}^\alpha \end{pmatrix} \begin{pmatrix} 0^{ij} & (m_D^T)^{i\beta} \\ m_D^{\alpha j} & M^{\alpha\beta} \end{pmatrix} \begin{pmatrix} \nu_L^j \\ (N^\beta)^c \end{pmatrix} \tag{22}$$

For $M \gg m_D$, this matrix gives approximate Majorana mass terms for the left-handed neutrinos of the form

$$m^{ij} \approx -(m_D^T)^{i\alpha} (M^{-1})^{\alpha\beta} m_D^{\beta j} \tag{23}$$

and the mass matrix becomes approximately block diagonal, with the heavy right-handed neutrinos effectively decoupling from the light left-handed neutrinos.

An advantage of the Majorana case is that the presence of the heavy right-handed Majorana neutrinos introduces a seesaw mechanism, in which the left-handed neutrino masses are suppressed by M^{-1} as seen in Eq. (23). As such, the small left-handed neutrino masses may be achieved without very small Yukawa couplings if the right-handed neutrinos are taken to be massive enough. Seesaw models of this type can also provide a successful mechanism for obtaining the baryon-antibaryon asymmetry as well as induce a negative Higgs mass through loop diagrams mediated by the right-handed neutrinos (for an appropriate choice of renormalization scale); however, the details of these mechanisms are outside the scope of this paper.

3 Unified theories

Although the only confirmed application of spontaneous symmetry breaking in the Standard Model is the generation of particle mass through the Higgs mechanism, SSB may also be used in other contexts. In this section, we review how grand unified theories (GUT), in which the theory is described by a single gauge group at high energies, may be broken down to the SM gauge group through spontaneous symmetry breaking. Although there are a number of possible GUT-scale groups, experimental constraints and required particle content place strong limits on many of them. Here we will review the gauge groups $SU(5)$ and $SO(10)$,nand describe how the SM may be realized in these groups.

In order to be broken to the SM theory, the unified gauge group must contain $SU(3) \times SU(2) \times U(1)$. The size of the group and its particular representations determine whether it is able to produce the currently-known SM particle content alone or whether it will inherently include further particle content. The heavy right-handed neutrinos discussed in the previous section may be included as a singlet of the unified theory if the irreducible representations do not naturally include such a particle, and naturally appear in certain GUT groups within the irreps which describe the SM fermionic content.

To provide some relevant context for the discussion of the choice of representations of the unified gauge groups into which we place the SM particle fields, we review the transformation of the SM fields under $SU(3) \times SU(2) \times U(1)$. In particular, the left-handed quark fields transform as triplets under $SU(3)$ and doublets under $SU(2)$ with $Y = 1/6$, which we indicate compactly by $(3,2)_{1/6}$, while the right-handed quarks are triplets under $SU(3)$ and singlets under $SU(2)$, $(\bar{3},1)_{1/3,-2/3}$. The leptons are all singlets under $SU(3)$, and the left-handed leptons are doublets under $SU(2)$, $(1,2)_{-1/2}$, while the right-handed electron, muon, and tau are singlets under $SU(2)$, $(1,1)_1$. The discussion in the following sections are a summary of material in Refs. [6]-[9]

3.1 SU(5)

The first proposed unified gauge group was $SU(5)$, originally introduced by Georgi and Glashow [6]. This is the smallest possible gauge group which contains the SM gauge group and provides the correct particle content for the SM. The particle content of the SM is contained in the irreducible representations of $SU(5)$ as:

$$\bar{\mathbf{5}} \to (\bar{3},1)_{\frac{1}{3}} \oplus (1,2)_{-\frac{1}{2}} \tag{24}$$

$$\mathbf{10} \to (3,2)_{\frac{1}{6}} \oplus (\bar{3},1)_{-\frac{2}{3}} \oplus (1,1)_1 \tag{25}$$

from which we see that the right-handed down-type quarks and the left-handed leptons are contained in the $\bar{\mathbf{5}}$ representation of $SU(5)$, while the left-handed quarks, right-handed up-type quarks, and right-handed leptons are contained in the $\mathbf{10}$ representation. We can therefore describe one full generation of the fermionic SM particles in the combination of irreducible representations $\bar{\mathbf{5}} \oplus \mathbf{10}$. One may also include the right-handed neutrino as a singlet, $\mathbf{1} \to (1,1)_0$.

There are two instances of spontaneous symmetry breaking required to take $SU(5)$ to the $SU(3) \times U(1)_Q$ SM group. The $SU(5)$ gauge group may first be broken to $SU(3) \times SU(2) \times U(1)$ using a scalar with a vev proportional to

$$\Phi_0 = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix} \tag{26}$$

where Φ transforms under the adjoint representation $\mathbf{24}$ of $SU(5)$. In this case, the vacuum leaves the $SU(3) \times SU(2) \times U(1)$ generators unbroken. The unbroken SM gauge symmetry is then spontaneously broken to $SU(3) \times U(1)_Q$ by a Higgs scalar, which is placed into a $\mathbf{5}$ representation of $SU(5)$.

The $SU(5)$ model runs into some phenomenological issues with regard to proton decay. Firstly, the massive gauge bosons obtained from spontaneously breaking $SU(5)$ to the SM gauge group can mediate proton decay through quark-lepton couplings, as both quarks and leptons now live inside the same irreps of $SU(5)$. An estimate of the proton lifetime obtained after including the decays mediated by the heavy gauge bosons is given by

$$\tau_{proton} \approx \mathcal{O}(10^4)\frac{M^4}{m_p^5} \tag{27}$$

where M is the mass of the heavy gauge bosons. This estimate places a bound on the mass scale M of approximately $M > 2\times10^{15}$ GeV, which also gives a lower limit on the unification scale.

Additionally, the SM Higgs is an $SU(2)$ doublet and therefore is typically included in a **5** representation of $SU(5)$, as noted above. This representation will also include an $SU(3)$ triplet, scalar lepto-quarks, which can mediate proton decay. Ensuring that this triplet does not induce a rate of proton decay in conflict with experiment requires it to be very massive. Obtaining a Higgs mass scale on the order of the weak scale as well as a heavy triplet from the same representation requires a high degree of fine-tuning; this issue is known as the doublet-triplet problem. Due to these considerations, the $SU(5)$ model requires heavy modifications or fine-tuning to remain viable.

One well-known extension of the basic $SU(5)$ model is the supersymmetric $SU(5)$ model [10], as minimal SUSY successfully achieves gauge-coupling unification at a scale of 10^{16} GeV. This model can provide some resolution to the doublet-triplet problem; however, the supersymmetric theory also runs into its own issues with proton decay, this time enhanced by Higgsinos.

3.2 SO(10)

While the $SU(5)$ theory provides an appealing structure through which we can describe the SM content in a single gauge group, the fermionic content is contained in two different irreps of the $SU(5)$ group. The group $SO(10)$, on the other hand, can describe all of the fermionic fields in a single representation. In particular, the spinorial **16** representation contains all of the SM fermion fields of a single generation; this also includes a right-handed neutrino as the 16th field. This structure therefore predicts one right-handed neutrino N^i for each of the SM generations.

There are multiple possible breaking patterns for $SO(10)$ to reduce to the SM gauge group. The spinoral rep **16** contains the $SU(5)$ rep $\bar{\mathbf{5}} \oplus \mathbf{10}$, so one particular example is breaking to $SU(5) \times U(1)$. Note that because $SO(10)$ is a rank 5 group, while $SU(5)$ is rank 4, there is an additional generator and therefore an additional gauge boson. The Higgs is placed in a **10** rep of $SO(10)$; the single Yukawa coupling allowed by $SO(10)$ is two fermion-matter **16**-plets coupled to the Higgs **10**. With this structure, the resulting masses do not reproduce the experimentally-measured values. Introducing further Higgs multiplets can modify the theory to accurately reproduce fermionic masses; **126** Higgs multiplets have been proposed as a solution. However, such Higgs multiplets appear unnatural.

The unbroken $SO(10)$ theory also does not allow a Majorana mass term for the right-handed neutrino, as it does not transform as a singlet under $SO(10)$. In this case, the mass scale of the right-handed neutrino might be expected to be near the scale at which the unified group is spontaneously broken; this scale is generally expected to be higher than the mass scale at which the seesaw mechanism successfully produces reasonable values for the masses of the left-handed neutrinos. The model can, however, be modified to give lower masses for the N^i.

4 Conclusion

In this paper, we have reviewed the applications of spontaneous symmetry breaking to the Standard Model. In particular, we have first focused on the Higgs mechanism and its subtleties in the quark sector, as well as possible realizations of neutrino mass mechanisms. The misalignment of quark mass eigenstates and weak eigenstates have been shown to introduce quark flavor-changing interactions through the charged weak current. The two possible cases for anti-neutrinos have been introduced, and we have reviewed proposed mass mechanisms in each case. While the Dirac neutrino terms may be easily introduced with a structure similar to the quark sector, the Dirac case has issues with the scale of the Yukawa couplings. Meanwhile, the Majorana case provides interesting solutions to additional problems in particle physics. In particular, the Majorana neutrino case can naturally explain the very small left-handed neutrino masses, provide a mechanism for baryon-antibaryon asymmetry through leptogenesis, and induce a negative Higgs mass through loop renormalizations. While some form of the Higgs mechanism has been confirmed by the discovery of the SM Higgs at the LHC, there are a number of interesting aspects of the mechanism which are still yet to be determined. The specifics of the Higgs sector may provide some answers or insight into other outstanding problems.

The second part of this paper has focused on the use of spontaneous symmetry breaking in recovering the SM gauge group from unified gauge groups at higher energy scales. We have introduced the proposed groups $SU(5)$ and $SO(10)$, and described the representations within which one may place the matter content of the SM. The issues of proton decay have been found to place strong limits on the $SU(5)$ model; in particular, the doublet-triplet problem introduces a significant issue for the non-SUSY $SU(5)$ model. The introduction of supersymmetry can provide some resolution to this issue, but the SUSY model runs into its own issues with over-predicting the rate of proton decay. Within the $SO(10)$ framework, we can place all matter content into a single representation, which provides an appealing unification; however, the specifics of the Higgs and its representation under $SO(10)$ raises its own problems.

Spontaneous symmetry breaking provides a rich framework through which one can approach many problems in high energy physics. Here we have seen here two very different motivations for introducing spontaneous symmetry breaking, and have encountered an elegant (if occasionally phenomenologically-troubled) solution in both cases. It seems likely that the SM Higgs mechanism will not turn out to be the only case in which we see an implementation of SSB in particle physics.

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