Problem Set 6 - Final

Physics 445

Due June 3

Some abbreviations: P&S - Peskin & Schroeder

Please do not discuss this problem set with other students. I want you to work on this individually. You can consult references.

1. Let's explore the beta function of non-abelian gauge theory in analogy with the abelian case we discussed in lecture. Take $SU(N)$ gauge theory with N_F fermions transforming in some representation of the gauge group. The number N_F is the number of flavors of fermion; each fermion transforms in the same representation of the gauge group. Let us also assume that each fermion has the same mass m. The gauge-fixed Lagrangian takes the form:

$$\mathcal{L} = -\frac{1}{4}\left(F^a_{\mu\nu}\right)^2 - \frac{1}{2\xi}\left(\partial^\mu A^a_\mu\right)^2 + \sum_{I=1}^{N_F} \bar{\psi}_I \left(i\not{D} - m\right)\psi^I + \text{ghosts}. \tag{0.1}$$

(i) What ghost terms are needed to complete this action?

(ii) Write down Feynman rules for this theory with general ξ.

(iii) Let's use Feynman gauge, $\xi = 1$, from now on. Compute the vacuum polarization, which is the renormalization of the gluon propagator, at one-loop using dimensional regularization.

(iv) Introduce renormalized fields and Z parameters as we did for QED. Compute the 1-loop renormalized Lagrangian using dimensional regularization.

(v) The charge is the coefficient of the $A_\mu\bar{\psi}\psi$ coupling. Let us call this coefficient g. Define $\alpha_s = \frac{g^2}{4\pi}$ in analogy with our QED discussion. Compute the 1-loop beta function and solve for the scale dependence of α_s.

(vi) Now specialize to $N = 2$. Imagine our fermions transform in the fundamental representation of the gauge group. How does α_s behave at low and high-energies as a function of N_F?

(vii) Lastly generalize this discussion of the behavior of α_s to fermions in a general representation of the gauge group $SU(2)$. We can label a general representation by "spin"-j in analogy with representations of the rotation group.