Hořava - Witten Theory

Piyush Kumar[†]

[†]*Enrico Fermi Institute and Department of Physics*
University of Chicago, Chicago, IL 60637, USA

email : piyush@theory.uchicago.edu

Abstract

This paper gives a detailed review of the work done by Hořava and Witten , who proposed that the strongly coupled limit of the $E_8 \times E_8$ heterotic string on $\mathbf{R^{10}}$ is the same as the low energy limit of M theory on $\mathbf{R^{10}} \times \mathbf{S^1}/\mathbf{Z_2}$. This equivalence is then used to derive Heterotic-Type I duality from an eleven dimensional perspective. Lastly, applications of Hořava-Witten theory to phenomenology and cosmology are presented.

1 Introduction

An understanding of any physical system requires dealing with interactions. If the interactions are weak, then perturbative expansion methods are reliable and help us analyze the physics. This approach has proved tremendously useful in the past for understanding a variety of systems - in atomic physics, in condensed matter and in particle physics.

However, perturbative methods cease to be reliable when interactions are strong. For example, there are many phenomena in particle physics such as quark confinement, Higgs mechansim and dynamical symmetry breaking, which arise from strong interactions and/or many degrees of freedom. In addition, there is a more serious objection to using perturbative methods. The perturbation series does not even define the theory. The series is at best asymptotic, not convergent, and so gives the correct qualitative and quantitative behavior at sufficiently small coupling only. It becomes useless as the coupling grows. In Quantum Field Theory, it is possible to give a nonperturbative definition of the theory by formulating it on a lattice with an intrinsic cutoff on the path integral.

Since String theory contains Quantum field theory, it also exhibits the above mentioned nonperturbative phenomena as well as a variety of new ones. However, unlike QFT, a totally nonperturbative formulation of String theory is missing, mainly because of the fact that string theory also has to deal with gravity. Nevertheless, during the last ten years or so, methods based on supersymmetry have revolutionized the understanding of String theory, particularly its strong coupling behavior. There is very strong evidence that all the five known string theories are limits of a single theory, which is eleven dimensional [1]. It is known as M theory. This revolution has been dubbed the "Duality revolution", because it indicates a duality between the weakly coupled limit of one theory to a strongly coupled limit of another theory.

The purpose of this paper is to review the work of Hořava and Witten [2] who have shown that the strong coupling limit of $E_8 \times E_8$ Heterotic string on $\mathbf{R^{10}}$ is the same as the low energy limit of M theory on $\mathbf{R^{10}} \times \mathbf{S^1}/\mathbf{Z_2}$.

The paper is organized as follows. Section 2 lists some of the most important dualities in string theory since we would be using the results for various purposes. Section 3 deals with M theory basics and maps out the strategy to provide convincing evidence for the stated conjecture. Sections 4, 5 and 6 analyze each of the three strategies in detail and also shed light on the structure of M theory as an intrinsically quantum theory. In Section 7, we try to arrive at the conjectured Heterotic-Type I duality from an eleven dimensional perspective. Section 8 deals with applications of Hořava-Witten theory to phenomenology and cosmology.

2 Dualities - important results

Let us itemize some of the most important dualities which would be useful to us :

a) *Type IIB - SL(2,Z) duality*

The low energy limit of Type II B string theory (which is Type IIB supergravity) is invariant under an SL(2,R) symmetry which among other things, converts weak coupling to strong coupling. In the full string theory, an SL(2,Z) subgroup survives and thus relates the weakly coupled Type IIB superstring to strongly coupled Type IIB superstring [3], [4].

b) *Heterotic - Type I duality*

It has been shown [5]that the Heterotic SO(32) theory on $\mathbf{R^9} \times \mathbf{S^1}$ is dual to the Type I SO(32) theory on $\mathbf{R^9} \times \mathbf{S^1}$ with the radius and the coupling related as :

$$\lambda_h = \frac{1}{\lambda_I};\ R_h = \frac{R_I}{\lambda_I^{1/2}} \tag{1}$$

where λ is the coupling and R is the radius of the circle.

c) *Type IIA - M Theory duality*

It can be shown that a collection of n D0-branes in Type IIA string theory is an ultra-short BPS multiplet of bound states with mass :

$$n\tau_0 = \frac{n}{\lambda^{2/3}} \tag{2}$$

This is an exact result, so as the coupling becomes large, the states become light and the spectrum approaches a continuum. This implies that as λ becomes large, the physics is described by an eleventh dimension of radius $R = \lambda^{2/3}$ [1]. This eleven dimensional quantum theory is known as M theory. Thus strongly coupled Type IIA string theory on $\mathbf{R^{10}}$ is the same as the low energy limit of M theory on $\mathbf{R^{10}} \times \mathbf{S^1}$.

3 M Theory basics

The low energy limit of M theory is eleven dimensional supergravity. The gamma matrices $\Gamma^I,\ I=1,...,11$ form a clifford algebra and also obey :

$$\Gamma^1\,\Gamma^2\,....\Gamma^{11} = 1. \tag{3}$$

It is reasonable to assume that M theory is well defined on orbifolds just as string theory is, with extra massless modes arising on fixed points. Suppose, we study M theory on a specific orbifold $\mathbf{R^{10}} \times \mathbf{S^1}/\mathbf{Z_2}$. On $\mathbf{R^{10}} \times \mathbf{S^1}$, M theory is invariant under supersymmetry generated by an arbitrary constant spinor. Moding out by Z_2 only leaves half of the supersymmetry invariant. The spinors ϵ can be chosen such that $\Gamma^{11}\epsilon = \epsilon$. Together with (3), we get :

$$\Gamma^1\,\Gamma^2....\Gamma^{10}\epsilon = \epsilon. \tag{4}$$

Thus, M theory on $\mathbf{R^{10}} \times \mathbf{S^1}/\mathbf{Z_2}$ reduces at low energies to a ten dimensional Poincare-invariant supergravity with one chiral supersymmetry. There are three known string theories with such a low energy structure - a) the $E_8 \times E_8$ heterotic string, b) Type I SO(32) string and (c) Heterotic SO(32) string. As will be shown later, by three separate convincing arguments, that M theory on $\mathbf{R^{10}} \times \mathbf{S^1}/\mathbf{Z_2}$ reduces to $E_8 \times E_8$ heterotic string for small radius. The three arguments are based on *space-time gravitational anomalies*, *strong coupling behavior* and *world-volume gravitational anomalies.*

4 Spacetime Gravitational Anomalies

The gravitational anomalies of M theory on $\mathbf{R^{10}} \times \mathbf{S^1}/\mathbf{Z_2}$ can be computed without knowledge of the full quantum M theory because anomalies can be computed just from the low energy structure. To find out if the theory is gravitational anomaly free, we need to know whether the effective action obtained by integrating out the gravitinos on $\mathbf{R^{10}} \times \mathbf{S^1}/\mathbf{Z_2}$ is invariant under diffeomorphisms.

On a smooth eleven dimensional manifold, it is known that there is no gravitational anomaly. However, on an orbifold, as above, there *is* an anomaly. This is because the Rarita-Schwinger field in eleven dimensions reduces in ten dimesnions to a massless chiral gravitino (anomalous) and an infinite sum of massive fields (anomaly free).

Under a spacetime diffeomorphism $\delta x^I = \epsilon\, v^I$, the change in effective action is given by :

$$\delta\Gamma = i\epsilon\int_{\mathbf{R^{10}}\times\mathbf{S^1}/\mathbf{Z_2}} d^{11}x\sqrt{g}\,v^I(x)\,W_I(x), \tag{5}$$

where g is the eleven dimensional metric and $W_I(x)$ is a local function at x. This is due to the fact that the anomaly can be understood to result entirely from the failure of the regulator to preserve the symmetries, and so can be computed from short distances. The contribution to the anomaly can only come from the fixed hyperplanes at $x^{11} = 0; (\Sigma_1)$ and at $x^{11} = \pi; (\Sigma_2)$ because on a smooth point in the orbifold, there is no anomaly. So (5) can be written as :

$$\delta\Gamma = i\epsilon\int_{\Sigma_1} d^{10}x\sqrt{g_1}\,v^I(x)\,W_I^{(1)}(x) + i\epsilon\int_{\Sigma_2} d^{10}x\sqrt{g_2}\,v^I(x)\,W_I^{(2)}(x) \tag{6}$$

where g_1 and g_2 are on Σ_1 and Σ_2 respectively. From symmetry, it is clear that $W_I^{(1)}(x) = W_I^{(2)}(x)$ symbolically. By choosing an arbitrary metric on $\mathbf{R^{10}}$ and the standard metric on $\mathbf{S^1}/\mathbf{Z_2}$, it can be shown easily that the two contributions from the above equation each give one-half of the standard ten dimensional result.

So, if M theory is to make sense on orbifolds, then there must be additional massless modes on the fixed hyperplanes, analogous to twisted sector modes in string theory on the fixed points. These extra massless modes, which have to be ten dimensional vector supermultiplets, can cancel the gravitational anomaly.

In ten dimensions, the anomaly is described by a formal twelve form $I_{12}(R, F_1, F_2) = A(R) + B(R, F_1) + B(R, F_2)$, which is a sixth order homogenous polynomial in the Riemann tensor R and the field strengths F_1 and F_2 on the two planes. $A(R)$ is the contribution of the supergravity multiplet, while $B(R, F_i)$ is the contribution of the ith vector multiplet. The irreducible part of A(R) - trR^6, can be cancelled by the addition of 496 vector multiplets, so that the possible gauge groups in ten dimensional N=1 superstring theory have dimension 496. From the eleven dimensional point of view, we are in the same situation except that the standard anomaly is divided equally between the two fixed hyperplanes. Thus, we must have 248 vector multiplets propagating on each hyperplane. This singles out $E_8 \times E_8$, with one E_8 propagating on each hyperplane.

So, we have :

$$\begin{aligned} I_{12}(R, F_1, F_2) &= \hat{I}_{12}(R, F_1) + \hat{I}_{12}(R, F_2) \\ \hat{I}_{12}(R, F) &= \frac{1}{2}A(R) + B(R, F) \end{aligned} \tag{7}$$

Till now, we have only taken care of the irreducible part of the anomaly. The reducible part of the anomaly also needs to be cancelled. For the case of ten dimensional perturbative heterotic string, a factorization of the twelve form into a four form and an eight form is required. For the present case, it can be shown that the anomaly twelve form on each hyperplane factorizes similarly, i.e $\hat{I}_{12} = \hat{I}_4 \hat{I}_8$, where $\hat{I}_8$ is a eight form written as a quartic polynomial in R and F_i and $\hat{I}_4 = trR^2 - trF_1^2 - trF_2^2$. The above factorization is crucial to permit an extension of the *Green-Schwarz* mechanism in ten dimensions to eleven dimensional manifolds with boundary.

The Green-Schwarz mechanism in String theory in ten dimensions depends on the existence of a two form field B whose field strength obeys $dH = I_4$. There are also Green-Schwarz terms present in string theory, but not in minimal low energy supergravity, given by :

$$L = \int B \wedge I_8 \tag{8}$$

Minimal classical ten dimensional supergravity is anomaly free because the anomalous fermion loops as well as Green-Schwarz interaction terms are both absent, while string theory is anomaly free because both the fermion loops and Green-Schwarz terms are present and the anomaly cancels.

In eleven dimensional supergravity, we have a three form field C whose field strength is a four form G. On a smooth eleven-manifold, it obeys the usual Bianchi identity dG=0. The analog of $dH = I_4$ in eleven dimensions will be that dG is a five form supported at the Z_2 fixed points. It turns out that :

$$dG = -\frac{3\sqrt{2}}{2\pi}(\frac{\kappa}{4\pi})^{2/3}\delta(x^{11})\, \hat{I}_4 \tag{9}$$

Green-Schwarz terms in eleven dimensional supergravity

In ten dimensions, the Green-Schwarz mechanism makes no general prediction about the form of $\hat{I}_8$. In eleven dimensions, however, the form of $\hat{I}_8$ can be predicted. The are two terms contributing to $\hat{I}_8$. One is the $\int C \wedge G \wedge G$ interaction of eleven dimensional supergravity. This interaction is involved in cancelling gauge anomalies. To cancel gravitational anomalies, there is another interaction - $\int_{M^{11}} C \wedge X_8(R)$, with X_8 an eight form constructed as a quartic polynomial in the Riemann tensor. This interaction can be deduced in two ways :

- By dimensionally reducing on S^1, it reduces to a $\int B \wedge X_8$ interaction which can be computed as a one-loop effect in Type IIA string theory. Since the calculation has no dilaton dependence, it can be extrapolated to strong coupling and hence to eleven dimensions.

- The same coupling is also needed to cancel the $M5$-brane world-volume anomalies which is crucial for the existence of $M5$-branes in the theory.

The two methods also agree on the form of $\hat{I}_8$. The success of this prediction is another convincing evidence that eleven dimensional supergravity on a manifold with boundary is indeed related to the $E_8 \times E_8$ heterotic string.

Gauge coupling constant, Classical and Quantum Consistency

Assuming that the above proposal is correct, it has implications for the structure of the theory. As we have already seen, one of the implications is that there must exist a supersymmetric coupling of ten dimensional vector supermultiplets on the boundary of the eleven-manifold to the eleven dimensional supergravity multiplet propagating in the bulk. From the gauge coupling λ and the gravitational coupling κ, we can form a dimensionless number $\eta = \lambda^6/\kappa^4$. We need to know what controls the value of η. Since eleven dimensional supergravity has no scalar field, the strong coupling limit of $E_8 \times E_8$ heterotic string, if it does have the eleven dimensional interpretation, must give a *definite* value of η. In fact, by looking rigorously at gauge and gravitational anomalies, we can determine η, $\eta = 128\,\pi^5$.

Another important implication is that on a manifold with boundary, the Green-Schwarz terms are already present at the classical level. This means that the classical Lagrangian with vector multiplets is *not* gauge invariant. This situation is strikingly different from perturbative string theory, where the Green-Schwarz term arises at one-loop level. Thus we have gauge invariance both classically (leaving out the anomalous one-loop fermion diagrams and the Green-Schwarz terms) and quantum mechanically (including both of them). Also, since the gauge kinetic energy is higher order compared to the gravitational kinetic energy, one must ignore the gauge fields to obtain a fully consistent classical theory. Otherwise, gauge invariance will fail classically and quantum anomalies will be needed to compensate for the failure.

The gauge anomalies that arise at the classical level give an indication (among many others) that M theory makes sense *only* at the quantum level.

5 Strong Coupling

To make a quantitative relation between M theory on $\mathbf{R^{10}} \times \mathbf{S^1}/\mathbf{Z_2}$ and $E_8 \times E_8$ heterotic string, we can compare the predictions of the two theories for the low energy effective action of the supergravity multiplet in ten dimensions. This gives the same result as in [1]:

$$R = \lambda^{2/3} \tag{10}$$

that one finds between M theory on $\mathbf{R^{10}} \times \mathbf{S^1}$ and Type IIA string theory (mentioned in section 2). Thus we get a candidate for the strong coupling behavior of the $E_8 \times E_8$ heterotic string. There is also another reason that if M theory on the above orbifold is related to one of the three ten dimensional string theories mentioned in section 3, then it must be the $E_8 \times E_8$ heterotic string. As we have seen in section 2, there is a very strong evidence that strongly coupled Type I string is weakly coupled Heterotic SO(32) string and vice versa. This means that we must relate the orbifold to the $E_8 \times E_8$ theory, whose strong coupling behavior was unknown previously.

6 Extended Membranes and Membrane world-volume anomalies

The third and last piece of evidence concerns extended membrane states in M theory after further compactification on $\mathbf{R^9} \times \mathbf{S^1} \times \mathbf{S^1}/\mathbf{Z_2}$. However, it should be made clear that this does *not* mean that M theory *is* a theory of membranes; it only means that M theory describes membrane states, among other things.

The simplest example of topologically stable membrane states is that of compactification of M theory on $R^9 \times S^1 \times S^1$. The solution is $x^2 = ... = x^9 = 0$ with x^1 as time and x^{10} and x^{11} as the two periodic variables. This solution is topologically stable and is invariant under half of the supersymmetries, i.e.

$$\Gamma^1\Gamma^{10}\Gamma^{11}\epsilon = \epsilon \tag{11}$$

which means that they are BPS saturated states. For the orbifold $R^9 \times S^1 \times S^1/Z_2$, we assume that the classical solution $x^2 = ... = x^9 = 0$ is still allowed. This means that the membranes of M theory can have boundaries that lie on fixed hyperplanes. The surviving supersymmetries correspond to spinors with $\Gamma^{11}\epsilon = \epsilon$. The spinors also obey the original condition (11), so we have $\Gamma^1\Gamma^{10}\epsilon = \epsilon$. Thus, out of the spinors which transform as $\mathbf{16}$ of $SO(1,9)$, or as $\mathbf{8}'_+ \oplus \mathbf{8}''_-$ of $SO(1,1)\times SO(8)$, only those transforming as $\mathbf{8}'_+$ survive. The massless modes on the membrane world-volume are worldsheet bosons $x^2, x^3, ..., x^9$ and worldsheet fermions transforming as $\mathbf{8}''_-$. Thus we see that we recover the worldsheet structure of the heterotic string : left and right moving bosons transforming in the $\mathbf{8}_v$ and right-moving fermions transforming in $\mathbf{8}''$. However, this does not imply that the string theory related to the M theory orbifold is heterotic, because Type I theory also has the same worldsheet structure.

To really decide in favor of the $E_8 \times E_8$ heterotic string, we need to consider membrane world-volume anomalies. By looking at them in detail, we can unambiguously infer that M theory on the given orbifold has $E_8 \times E_8$ heterotic string as its weak coupling limit.

7 Heterotic - Type I duality from M theory

Schwarz [3] and Aspinwall [4] have shown the existence of $SL(2, Z)$ duality of ten dimensional Type IIB string theory (mentioned in section 2), which follows from the spacetime diffeomorphism symmetry of M theory on $R^9 \times T^2$. Similarly, we can hope to recover heterotic- Type I duality from the classical symmetries of M theory on $R^9 \times S^1 \times S^1/Z_2$.

The Type I theory can be thought of as a generalized Z_2 orbifold of Type IIB theory, which can be obtained by reversing world sheet parity. On compactifying on $R^9 \times S^1$, Type IIB theory is T-dual to Type IIA theory. So, Type I theory, which is a Z_2 orbifold of Type IIB theory, is T-dual to a certain Z_2 orbifold of Type IIA theory. This is called the Type IA theory. The twisted states in this theory are open strings that have their endpoints at $x^{10} = 0$ and at $x^{10} = \pi$. They must carry SO(16) Chan- Paton factors to cancel anomalies, and since the end points have to be treated symmetrically, the gauge group is $SO(16) \times SO(16)$.

Lets now try to relate the eleven dimensional theory to the proposed heterotic-Type I duality with SO(32) gauge group. On one side, we start with M theory on $R^9 \times S^1/Z_2 \times S^1$, and interpret it as Type IIA theory on $R^9 \times S^1/Z_2$, since one associates Type IIA theory on a space $\mathcal{M}$ with M theory on $\mathcal{M} \times S^1$. This, in turn, is the T-dual of Type I theory on $R^9 \times S^1$ with an $SO(16) \times SO(16)$ vacuum. On the other side, we could start with M theory on $R^9 \times S^1 \times S^1/Z_2$, and interpret it as the $E_8 \times E_8$ heterotic string on $R^9 \times S^1$. By turning on a wilson line, we could break the gauge group to $SO(16) \times SO(16)$. This is T-dual to Heterotic $SO(32)$ theory with an $SO(16) \times SO(16)$ vacuum. Then, we can compare the two sides using Heterotic - Type I duality, which should give the same relation as expected from the eleven dimensional point of view.

Side 1

Start with M theory on $R^9 \times S^1/Z_2 \times S^1$, with radii R_{10} and R_{11} for the two circles. Interpret this as Type IA theory, a Z_2 orbifold of Type IIA theory on $R^9 \times S^1$, which is T-dual to Type I theory in a vacuum with unbroken $SO(16) \times SO(16)$. The relation between R_{10} and R_{11} and Type IA parameters with string coupling λ_{IA} and radius R_{IA} of S^1 can be computed by comparing the low energy actions for the supergravity multiplet. We find as in [1] and also in section 2:

$$R_{11} = \lambda_{IA}^{2/3}; \;\; R_{10} = \frac{R_{IA}}{\lambda_{IA}^{1/3}} \tag{12}$$

By a T-duality transformation to Type I with unbroken $SO(16) \times SO(16)$, we get :

$$R_{11} = (\frac{\lambda_I}{R_I})^{2/3}; \;\; R_{10} = \frac{1}{R_I^{2/3}\lambda_I^{1/3}} \tag{13}$$

using the formulas $R_I = 1/R_{IA}$ and $\lambda_I = \lambda_{IA}/R_{IA}$.

Side 2

Start with M theory on $R^9 \times S^1 \times S^1/Z_2$, with the radii of the last two factors denoted as R'_{10} and R'_{11}. This should be related to $E_8 \times E_8$ heterotic string on $R^9 \times S^1$, with the M theory parameters being related to the $E_8 \times E_8$ heterotic string coupling λ_{E8} and radius R_{E8} as :

$$R'_{11} = \lambda_{E8}^{2/3}; \;\; R'_{10} = \frac{R_{E8}}{\lambda_{E8}^{1/3}} \tag{14}$$

just like (12) and obtained in the same way too. We then turn on a Wilson line and make a T-duality transformation to an $SO(32)$ heterotic string with parameters λ_h and R_h related to those of the $E_8 \times E_8$ theory by $R_h = 1/R_{E8}$ and $\lambda_h = \lambda_{E8}/R_{E8}$. Thus, we get the analog of (13) :

$$R'_{11} = (\frac{\lambda_h}{R_h})^{2/3}; \;\; R'_{10} = \frac{1}{R_h^{2/3}\lambda_h^{2/3}} \tag{15}$$

Comparison

We can now compare the two sides by the proposed Heterotic-Type I duality, mentioned in section 2, according to which these theories are identified with :

$$\lambda_h = \frac{1}{\lambda_I}; \quad R_h = \frac{R_I}{\lambda_I^{1/2}} \tag{16}$$

Comparing (13) and (15), we see that :

$$R_{10} = R'_{11}; \quad R_{11} = R'_{10} \tag{17}$$

Thus, we see that under the above sequence of operations, the natural symmetry in eleven dimensions becomes standard Heterotic-Type I duality.

8 Applications - Phenomenology and Cosmology

The set-up of Hořava-Witten theory is quite conducive for applications to phenomenology as well as cosmology. The first thing we need to make a realistic model is to come down in dimensions. The set-up of Hořava-Witten suggests the following picture - six of the ten dimensions are compactified leaving a five dimensional theory which is the product of a smooth four dimensional manifold with the orbifold S^1/Z_2. Thus we have a four dimensional theory at the boundary planes of the orbifold. In fact, Witten has shown that such a compactification is possible for a deformed Calabi-Yau threefold [6]. One finds that the radius of the Calabi-Yau threefold is smaller than the orbifold radius by a factor of ten or so. Since the GUT scale ($\sim 10^{16}$ GeV) is set by the size of the Calabi-Yau, this implies that below the unification scale, there is a regime where the universe appears five dimensional.

This arrangement has some distinct phenomenological advantages over the perturbative $E_8 \times E_8$ heterotic string. By changing the size of the orbifold l, one can arrange for a unification of gauge and gravitational couplings. Choosing a large value of l relative to the eleven dimensional Planck length also justifies the use of field theoretic degrees of freedom.

The five dimensional theory splits into a bulk $N = 1, d = 5$ supersymmetric theory with the gravity multiplet and the universal hypermultiplet and two four dimensional boundary theories which reside on the orbifold fixed hyperplanes. The fields on the boundary come from the $E_8 \times E_8$ gauge multiplet, which are $N = 1, d = 4$ gauge and chiral multiplets.

An "ideal case" scenario for such a model would be as follows : The internal six dimensional space and the orbifold evolve in time for a short period and then settle down to their "phenomenological" values while the non-compact dimensions continue to expand. Then, for late time, when all physical scales are much large than the orbifold size, the theory is effectively four dimensional and should in the "static" limit, provide a realistic supergravity model of particle physics. As can be shown, such realistic supergravity models originate from a reduction of the five dimensional theory on its domain wall vacuum state [7]. In addition, we could look for realistic cosmological solutions depending both on the orbifold coordinate and time [7], which should reduce in the static limit at late time to the domain wall solution that incorporates breaking of the remaining $N = 1, d = 4$ supersymmetry.

The five-dimensional effective action

The effective action for Hořava-Witten theory, obtained by compactifying on a Calabi-Yau threefold is derived in [8] for the universal zero modes, i.e. the five dimensional graviton supermultiplet and the breathing mode of the Calabi-Yau, along with its superpartners. Instead of writing down the effective action in full glory, let's just discuss the relevant fields arising from such a set-up.

We have the five-dimensional gravity supermultiplet with the metric $g_{\alpha\beta}$ and an abelian gauge field $\mathcal{A}_\alpha$ as the bosonic fields. The fields coming from the Calabi-Yau form the universal hypermultiplet in five dimensions. The bosonic fields in the universal hypermultiplet are the real scalar V (volume of the Calabi-Yau), the three form $C_{\alpha\beta\gamma}$ (dualized to give a scalar σ) and the Ramond-Ramond scalar ξ. Lets now consider the boundary theories. In M_5, the orbifold fixed planes constitute four dimensional hypersurfaces $M_4^{(i)}$, $i = 1, 2$. With the standard embedding in the reduction from eleven to five dimensions, there will be an E_6 gauge field $A_\mu^{(1)}$ and matter fields on $M_4^{(1)}$. On $M_4^{(2)}$, there is an E_8 gauge field $A_\mu^{(2)}$.

One of the most striking features of the action is that because of the bulk and boundary potential terms for the dilaton V, purely time dependent solutions of the theory don't exist. The same fact also precludes flat space from being a solution. The natural solution turns out to be a pair of parallel three-branes corresponding to orbifold planes. This pair of "domain walls" can be viewed as the "vacuum" of the five dimensional theory, in the sense that it provides the appropriate background for a reduction to the $d = 4, N = 1$ theory. This picture can then be put into the context of cosmology [7], and thus be made dynamical.

Supersymmetry Breaking

Hořava-Witten theory also provides an elegant possible mechanism for the breaking of supersymmetry. The gauge matter fields could be at strong coupling on one boundary, and supersymmetry could be broken spontaneously there. This would be a direct realization of *hidden sector supersymmetry breaking.* Supersymmetry breaking effects could then be communicated to the other boundary (where we live) by five dimensional fields. In fact Hořava has tried to make the this mechanism of communication explicit by exhibiting a term in the 11 dimensional Lagrangian which couples the gaugino condensate on the boundary to the 3-form gauge field C_{ABC} of the bulk supergravity theory [9]. This term has a perfect square structure :

$$\Delta L = \frac{-1}{12\kappa^2} \int d^{11}x \, (\partial_{11} C_{ABC} - \frac{\sqrt{2}}{16\pi} (\frac{\kappa}{4\pi})^{2/3} \, \bar{\chi} \Gamma_{ABC} \chi \, \delta(x^{11}))^2 \tag{18}$$

where χ is the ten dimensional gaugino. It was argued by Hořava that there is no supersymmetry conserving solution if the gaugino bilinear obtains a nonzero expectation value.

It is still not known whether such a mechanism is completely satisfactory. The origin of the squared delta function and the inclusion of such a term in a purely field-theoretic description of Hořava-Witten is not clear. People have tried to understand this mechanism by studying various simplified toy models [10].

9 Summary

This paper has tried to provide a reasonably detailed review of the original claim by Hořava and Witten that the strongly coupled limit of $E_8 \times E_8$ heterotic string is the same as the low energy limit of M theory on $\mathbf{R^{10}} \times \mathbf{S^1}/\mathbf{Z_2}$. We see that the structure of Hořava-Witten theory is an extremely elegant one which lends itself to a variety of phenomenological and cosmological applications. It has many advantages over approaches to model building from the weakly coupled $E_8 \times E_8$ heterotic string. However, constructing a completely satisfactory model of supersymmetry breaking and cosmology (with an inflationary phase) is still an open problem.

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